POLARIZED LIGHT

POLARIZED LIGHT

PRODUCTION AND USE

WILLIAM A. SHURCLIFF
Research Fellow in Physics, Harvard University

HARVARD UNIVERSITY PRESS
Cambridge, Massachusetts, 1962

Distributed in Great Britain
by Oxford University Press, London

Library of Congress Catalog Card Number 62–11405
Printed in the United States of America

PREFACE

Because polarized light is being used increasingly—by physicists, chemists, biologists, metallurgists, mineralogists, mechanical engineers, and electronics engineers—the need for a serious book on the production and use of polarized light has become increasingly evident. Every year hundreds of additional articles dealing with polarized light appear in various scientific journals. New applications are constantly being reported. But there has been no book to delineate the central concepts, to indicate a comprehensive terminology, to compare the different types of polarizers, and to present the rules governing the combinations of polarizers and retardation plates. There has been no careful review of the hundreds of kinds of applications, and no substantial bibliography.

The most important event in the modern history of polarized light was the invention of the sheet-type polarizer, by Edwin Herbert Land in 1928. His invention of the microcrystalline species of sheet-type polarizer (J-sheet), and the later invention by Land and his associates of the molecular species (H-sheet, K-sheet, HR-sheet, and so forth), provided scientist and engineer with polarizers having almost every desirable feature. Nearly every branch of science has felt the impact of these inventions. Yet the technology of these modern polarizers has received scant mention in the scientific literature.

Four powerful tools for predicting the effects of polarizers, retardation plates, and so on have recently come into prominence, but have not been discussed in available textbooks in a serious, systematic way. The new tools are the Stokes vector, the Poincaré sphere, the Mueller

calculus, and the Jones calculus. They make it possible to calculate with ease the behavior of polarizer-retarder combinations that formerly seemed almost hopelessly complicated. Here we describe the tools in detail and illustrate their use. In addition, all the commonly required matrices of the Mueller and Jones calculi are listed, for ready reference.

The author's early training in polarization phenomena was acquired in the research laboratory directed by Dr. E. H. Land. The writings by Dr. Land and his colleagues Dr. Cutler D. West and Dr. R. Clark Jones established the foundations on which this book is based.

The author's debt to Dr. R. Clark Jones, the inventor of the Jones calculus, is immeasurable. The sections dealing with the Stokes vector, the Mueller calculus, and the Jones calculus could not have been written without long and painstaking coaching by him. The help received from E. S. Emerson and A. S. Makas in various practical aspects of polarizer technology has been of great value. Many other colleagues have helped, directly or indirectly, to make this monograph possible.

W. A. Shurcliff

Cambridge, Massachusetts
August 25, 1961

CONTENTS

ILLUSTRATIONS

TABLES

POLARIZED LIGHT

1 CONVENTIONAL DESCRIPTION OF POLARIZED LIGHT

1.1. Introduction. In this book the term *light* stands for *electromagnetic radiation.* Usually we have in mind the 400–700 mμ range (visual range), but often we include also the shorter-wavelength range (ultraviolet) and the longer-wavelength range (infrared). On some occasions we include also the x-ray and gamma-ray ranges and the radio range. The total range in which polarization plays a part covers more than sixty octaves.

Polarized light is one of nature's ultimates. A slender, monochromatic, *polarized* ray cannot be subdivided into simpler components: no simpler components exist. The process of analysis can advance no further.

There is much to be gained, however, by considering how a beam of polarized light behaves and how it may be depicted. When such a beam encounters a birefringent crystal, a dichroic film, or an oblique dielectric surface, a great variety of behaviors may result. The question is: Can we find, for the polarized beam, a representation so pertinent and so versatile that, merely by examining the representation, we can predict the outcome of any given encounter?

Fortunately, several highly successful representations have been invented. Some are pictorial, others mathematical. Some are well suited to solving simple problems, others are to be preferred when the problems are complicated.

To ask whether a given representation is "true" is futile. It must suffice that the representation assists ready recollection of the behavior and permits easy solution of the various problems encountered.

The present chapter reviews the classical (pictorial and wave-train) methods of representing polarized light. Chapter 2 considers certain more modern and more powerful methods.

Polarized light, besides being of interest *per se*, serves as a tool, or probe, for evaluating the properties of matter. The tool exhibits the ultimate in speed, and perhaps the ultimate in delicacy and convenience. It has the merit of being completely convertible; that is, the polarization form can be altered at will, with no loss in power and no increase in entropy flux. In many respects, polarized light, being the simplest kind of light, is easier to deal with than ordinary light: the physical manipulations may be cleaner, and the mathematical procedures for predicting the experimental outcomes are simpler. Physicists and chemists find that polarized light has uses far beyond those of unpolarized light. Biologists, astronomers, and engineers find that polarized light solves many problems that are otherwise insoluble. If light is man's most useful tool, polarized light is the quintessence of utility.

In preparing this book the author faced a major problem as to conventions. The crux of the problem was the large number of branches of optics that must be brought into one consistent family. Traditionally, users of saccharimeters and other polarimeters employ a certain set of sign conventions, persons dealing with dichroism employ certain conventions, and similarly for persons dealing with crystallography, wave theory, the Stokes vector, the Poincaré sphere, the Mueller calculus, and the Jones calculus. Ordinarily, the incompatibility of the various sets of conventions as to signs, handedness, etc., is unnoticed and unimportant. In this book, however, one universally self-consistent set of conventions is mandatory. Accordingly, some conflict with various lesser sets is unavoidable.

1.2. Classical Pictorial Specification of a Polarized Wave Train. The classical description of a polarized wave train is well known (see, for example, Ditchburn, D-10, and Jenkins and White, J-9), and needs only brief review here.

From the standpoint of classical physics, light consists of electromagnetic waves whose vibrations are transverse to the propagation direction. *Polarized* light is light whose vibration pattern exhibits *preference:* preference as to transverse direction, or preference as to the handedness associated therewith. Different kinds of preference are

indicated in Fig. 1.1. Each drawing, called a snapshot pattern, describes the monochromatic wave train at a single instant in time; the curve may be thought of as a smooth line joining the tips of a large number of vectors that indicate the directions and magnitudes of the *electric* field at various positions along the center line of the beam. The convention with respect to right and left circular polarization is easily remembered: right circular polarization is portrayed by means of a right-handed helix, such as the thread of a typical machine screw.

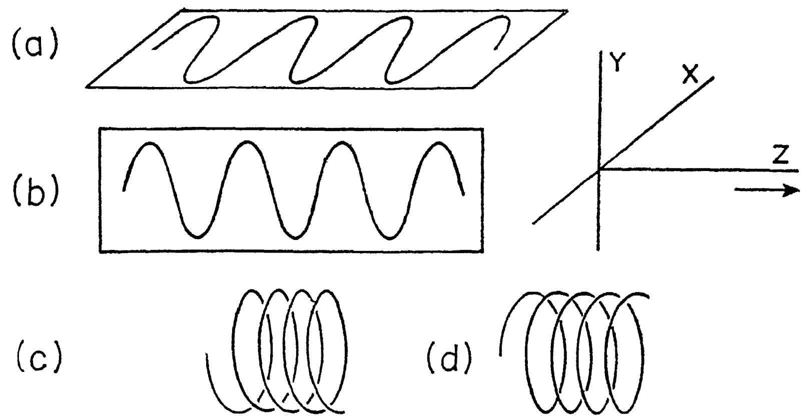


FIG. 1.1. Snapshot patterns of a horizontally traveling beam of monochromatic light that is polarized (*a*) horizontally, (*b*) vertically, (*c*) right circularly, and (*d*) left circularly.

(The reader will recall that a right-handed helix continues to appear right-handed no matter what the observer's viewpoint; consequently the present definition is free from ambiguity.) The pattern may be drawn with respect to a right-handed set of cartesian coordinates, Z being the direction of propagation, and X and Z being horizontal.

Workers in different fields (such as crystallography, theoretical physics, saccharimetry, radio technology) may employ conflicting definitions. The definitions used in this book are believed to represent the best compromise. Care has been taken to word the definitions clearly and to use them consistently.

One could, of course, deal with the magnetic, rather than the electric, vibration. When light is traveling in a vacuum or other isotropic medium, these two vibrations are orthogonal (perpendicular) and their magnitudes are always proportional to one another. To specify

one is tantamount to specifying both. The decision to concentrate on the *electric* vibration is conventional, and pays tribute to the dominant role of the electric vector in the more familiar absorption processes.

The *sectional pattern* (Fig. 1.2) is perhaps the most familiar of all the characterizations. A horizontally polarized beam is portrayed as a

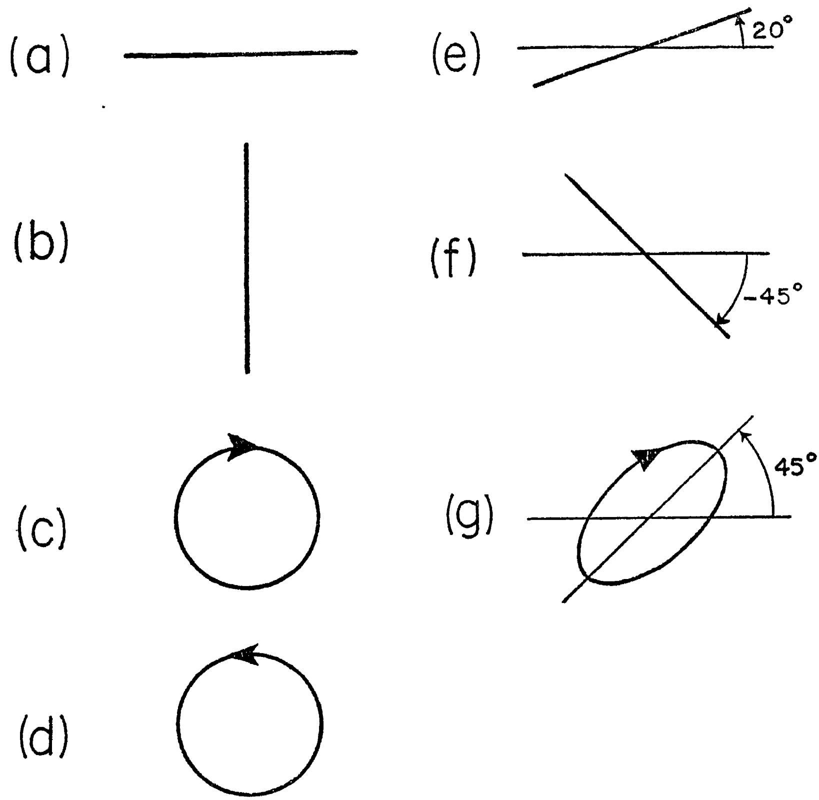


FIG. 1.2. Sectional pattern of a beam polarized (*a*) horizontally, (*b*) vertically, (*c*) right circularly, (*d*) left circularly, (*e*) linearly at 20°, (*f*) linearly at −45°, (*g*) right elliptically at 45°.

short horizontal line; vertical polarization is indicated by a vertical line. Right-circular polarization is portrayed by a circle having a clockwise sense. The sectional pattern may be thought of as an end view of the snapshot pattern, as seen by an observer who is situated in the path of the beam (specifically, far out on the Z-axis) and is looking toward the light source, which is at the origin of coordinates.

The clockwise sense of the circle describing right-circular polarization is consistent with the definition involving a *right-handed helix:* if a right-handed helix is moved bodily toward an observer (without rotation) through a fixed, transverse, reference plane, the point of intersection of helix and plane executes a *clockwise* circle.

The sectional patterns are easy to draw, even for light that is polarized elliptically. Also, they can embrace polychromatic light; however, we must then think of the scale of the pattern as changing more or less rapidly, depending on the frequency bandwidth of the beam; consequently the patterns (Fig. 1.3) cease to be of simple, closed type.

FIG. 1.3. Appearance of sectional pattern of an elliptically polarized beam having small but appreciable bandwidth, assuming observation times of (a) about 1 cycle, (b) about 2 cycles, (c) many cycles.

The general sectional pattern of a monochromatic beam — an ellipse — may be described with the aid of the terms defined in Fig. 1.4. The angle α (between the major semiaxis and the X-axis) is called the *azimuth* of the sectional pattern; $90° \geq \alpha \geq -90°$. The ratio b/a of the semiaxes is called the *ellipticity;* the symbol β may be used to represent arc tan b/a; $90° \geq \beta \geq -90°$. Ellipticity is used in preference to the eccentricity, which is defined as $(a^2 - b^2)^{1/2}/a$.

In some instances the ratio A_y/A_x is of interest; A_y is the maximum value of the Y-component of the electric vector, and A_x is the maximum value of the X-component. The angle $|\text{arc tan }(A_y/A_x)|$ will be

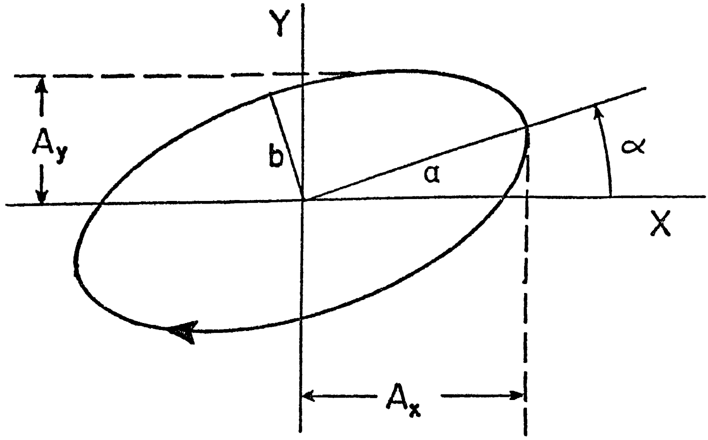


FIG. 1.4. Elliptically polarized light. In this example, $\alpha = 20°$, the ellipticity $b/a = 0.4$, and the handedness is clockwise, as judged by an observer situated far out on the Z-axis and looking backward toward the source, which is at the origin.

called R. When α is in the neighborhood of $\pm 45°$, or when the ellipse is very slender, the angles $|\alpha|$ and R are not very different, but under other circumstances they differ greatly.

Polarization Types and Forms. Linear polarization, circular polarization, and elliptical polarization may be referred to as the three *polarization types*. Obviously, the elliptical type includes the others as special cases; ellipticities of 0 and 1 correspond to linear and circular polarization respectively.

The linear type of polarization includes an infinite number of *polarization forms*, differing as to azimuth α. Circular polarization includes two forms, differing as to handedness. Elliptical polarization includes an infinite number of forms, differing as to azimuth, ellipticity, and handedness.

Orthogonal Forms. Two forms of linear polarization that differ by exactly 90° in azimuth are said to be orthogonal, assuming the directions of propagation to be the same (Fig. 1.5). Right- and left-circularly polarized beams are orthogonal. Two elliptically polarized beams are orthogonal if the azimuths of the major axes differ by 90°, the handednesses are opposite, and the ellipticities are identical.

Plane of Polarization. The expression *plane of polarization*, used by many authors, may be ambiguous. To some authors it means the plane containing the directions of propagation and of the electric

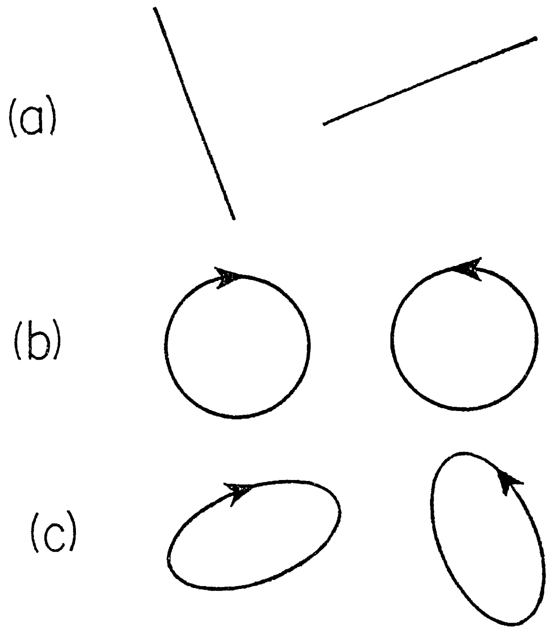


FIG. 1.5. Orthogonal pairs of beams polarized (*a*) linearly, (*b*) circularly, (*c*) elliptically.

vibration, while to others it means the plane containing the directions of propagation and of the *magnetic* vibration. Another drawback to the expression is that an experimenter can easily produce a number of beams that have the *same* plane of polarization yet *different* directions of electric vibration (Fig. 1.6). Likewise he can produce beams having *different* planes of polarization and the *same* direction of vibration.

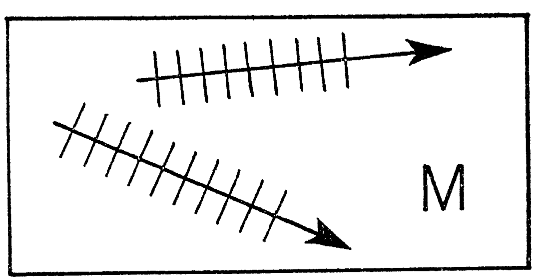


FIG. 1.6. Two beams having the same plane of polarization (plane *M*) yet different directions of electric vibration (indicated by hatch marks, all of which lie in plane *M*).

In this book these difficulties are avoided by the expedient of using terms descriptive of the key quantity, namely, the direction of the electric vibration. We refer to *linearly* polarized light and to the *direction of vibration*. We avoid expressions such as *plane polarized light*, and *plane of polarization*.

1.3. Mathematical Specification of a Polarized Wave Train. The mathematical specification of a wave train is explained in the standard textbooks on electromagnetic theory (D-10; A-1). Simple, monochromatic, linearly polarized trains of plane waves are propagated by means of transverse displacements varying sinusoidally with time and with position along the propagation direction. The magnitude ξ of the electric displacement **E** may be described by an expression such as

$$\xi = \sin(\omega t - 2\pi Z/\lambda),$$

where Z is the position along the axis of propagation, λ is the wavelength, ω is the angular frequency (2π times the ordinary frequency), and t is time.

To facilitate computations of certain sorts, one may introduce complex notation. To make the expressions more versatile, one may include a constant A, called the magnitude of the peak amplitude, and a quantity ϵ, called the epoch. The expression may take any of the following forms:

$$\begin{aligned} \xi &= Ae^{i\epsilon}e^{i\omega t}e^{-i2\pi Z/\lambda} \\ &= Ae^{i\epsilon}e^{i(\omega t - 2\pi Z/\lambda)} \\ &= Ae^{i(\epsilon + \omega t - 2\pi Z/\lambda)} \\ &= Ae^{i\phi}, \end{aligned}$$

where $\phi = \epsilon + \omega t - 2\pi Z/\lambda$. The real part of ξ represents the instantaneous magnitude of the electric vector **E** (at time t and position Z). The quantity $Ae^{i\phi}$ is called the complex amplitude. The quantity ϕ $[= \epsilon + \omega t - 2\pi Z/\lambda]$, is the phase angle at time t and position Z. The intensity of the beam depends on A, and is, of course, proportional to A^2.

The term *intensity* is used in this book in a number of ways. Sometimes it means the total power of the beam. On other occasions it means power per unit solid angle, or power per unit solid angle and per unit cross-sectional area. The intended meaning is usually made clear by the context. Formal definitions of intensity are presented by Chandrasekhar (C-8).

The direction of the electric vector does not appear in the equations, but may be specified separately — verbally or pictorially; or it may be specified by including unit vectors **i** and **j**, parallel to the X- and Y-axes respectively.

A circularly polarized beam of monochromatic light may be represented by a combination of two expressions, each having a complex magnitude of the form $Ae^{i\phi}$. One expression describes the vertical component (Y-component), and is written $A_y e^{i\phi_y}$; the other describes the horizontal component (X-component) and is written $A_x e^{i\phi_x}$. The amplitudes A_y and A_x are equal, and the phase angles ϕ_y and ϕ_x differ by 90°. If the difference $(\phi_y - \phi_x)$, called γ, is positive (90°), the light is *right*-circularly polarized. If $\gamma = -90°$, the light is left-circularly polarized.

In the general case, the Y- and X-components differ in amplitude, and γ may have any value; the general sectional pattern is, of course, an ellipse. If $180° > \gamma > 0°$, the handedness is right; if $-180° < \gamma < 0°$, the handedness is left. When $\gamma = 0°$, the pattern consists of a straight line (linear polarization), and when $|\gamma| = 90°$ the pattern is a circle.

To predict the outcome of adding two monochromatic polarized beams, one adds their instantaneous electric vectors. In general, the two vectors have different directions in real, three-dimensional space, different frequencies, and unrelated phases; hence the result of the addition is a complicated and not very useful expression. In the simple case in which both beams are linearly polarized and have the same frequency and same phase, the procedure is simply to add the two vectors representing the root-mean-square electric vibrations of the two beams (Ref. W-1, p. 27).

If the two linearly polarized beams differ in phase (by some constant amount), the procedure is more complicated; if the phase difference is 180°, and if the two beams have equal intensity, the combined beam will have zero intensity. When two *coherent* beams intersect at a slight angle, the phase relation varies, of course, from one point to another in the region of intersection (as explained in Refs. D-10 and B-43, coherent beams are beams whose phases have a fixed, or virtually fixed, relation to one another); consequently the combined beams will have high intensity at some points and low intensity at others, so that an *interference pattern* results.

If the beams are completely *incoherent*, a short-cut procedure is

available: merely add the intensities of the beams. The sum of the intensities is the intensity of the combined beam.

Later chapters make it clear that the combining of beams can usually be handled more simply by certain modern methods than by the classical equations presented above. The Stokes vector provides an ideal basis for treating the combining of incoherent beams; the Jones vector is eminently applicable to coherent beams. These vectors are discussed in Chapter 2.

1.4. Unpolarized Light. Defined operationally, an unpolarized beam is a beam that, when operated on by any elementary kind of energy-conserving device that divides the beam into two completely polarized subbeams, yields subbeams that have *equal power* (in a time interval long enough to permit the powers to be measured). Thus if a beam is to qualify as unpolarized, it must exhibit no long-term preference as to lateral direction of vibration or as to handedness.

Can a perfectly monochromatic beam qualify as unpolarized? Obviously it cannot. Such a beam necessarily has a perfectly regular wave train, and consequently has a very definite and steady sectional pattern. Thus it exhibits polarization. Almost perfectly monochromatic radio waves are a common occurrence, and are found, of course, to exhibit a high degree of polarization.

Visible light, however, always possesses an appreciable bandwidth. Accordingly, such a beam may include many different forms of polarization simultaneously. If an experimenter is unable to detect any preponderant azimuth or handedness, he will perforce regard the beam as being unpolarized. (This subject has been explored by Langsdorf and DuBridge, L-14, and by Birge and DuBridge, B-29.) In this book the expression "monochromatic light" often appears; usually it means light that is *roughly* monochromatic and has sufficient bandwidth that unpolarized behavior is not precluded.

At most moments, a beam of unpolarized light has, of course, a sectional pattern that is elliptical. Hurwitz (H-41) has computed the average value of ellipticity, which turns out to be tan $15°$ or 0.268.

No satisfactory way of describing unpolarized light pictorially has been found. To portray unpolarized light as a many-pointed star or asterisk is conventional, but without scientific merit; the portrayal fails to suggest the most prominent features of unpolarized light: its constantly changing, predominantly elliptical, character.

1.5. Partially Polarized Light. Since most light, whether of natural or artificial origin, is neither completely polarized nor completely unpolarized, the concept of *degree of polarization* is an important one. The quantity is defined in terms of the components into which the beam may be divided.

There are two useful ways of mentally dividing a beam into components. One method employs a "polarized–unpolarized" dichotomy, the other employs a "vibration-form" dichotomy. In employing the *polarized–unpolarized* dichotomy, one divides the given beam into a component C_a that is completely polarized and a component C_b that is unpolarized and has no long-term phase relation (no coherence) with C_a. It is a well-established fact that the outcome of the division process is unique: there is only one possible pair C_a and C_b. Depending on whether C_a is linearly, circularly, or elliptically polarized, the partially polarized beam in question is called partially linearly, partially circularly, or partially elliptically polarized.

In employing the *vibration-form* dichotomy, one divides the beam into that pair of completely and orthogonally polarized components that have the maximum difference in intensity. The more intense component may be called the *dominant component* C_d and the less intense component may be called the *inferior component* C_i. It may be shown that the two components are mutually incoherent; no long-term correlation between their phases exists.

The polarized–unpolarized dichotomy is useful in a variety of problems involving partial polarizers, retarders, and so forth. For example, if a partially polarized beam strikes a retarder, the unpolarized component is entirely unaffected thereby and accordingly the investigator is left free to concentrate his attention on the polarized component. The dichotomy has one notable limitation: no known device can perform the indicated analysis. (However, the converse process is easily achieved: it is an easy matter to combine a polarized beam and an unpolarized beam — locally, at least: one merely causes them to intersect at a small angle. By using this process, one can demonstrate the validity of the rationale under discussion.)

The vibration-form dichotomy is useful in problems involving those polarizers and polarizing beam splitters that employ birefringence. These devices are capable of accomplishing just such a dichotomy (within certain practical limitations).

The degree of polarization V is now easily defined. If the intensities

of components C_a, C_b, C_d, and C_i are called I_a, I_b, I_d, and I_i respectively, V is defined by either

$$V = \frac{I_a}{I_a + I_b}$$

or

$$V = \frac{I_d - I_i}{I_d + I_i}.$$

An author may wish to use a term that describes the incompleteness, or shortage, of polarization. The term *polarization defect*, defined as $I_i/(I_d + I_i)$, or its equivalent $(1 - V)/2$, may be used (S-11).

On some occasions, the concept of *equivalent degree of linear polarization* is a useful one. A beam that is 100-percent elliptically polarized and has a very small ellipticity might be judged to be not *completely, elliptically* polarized, but *incompletely, linearly* polarized; the naively arrived at result (equivalent degree of linear polarization) provides a figure that is useful in certain applications.

Spectrally variegated polarization is, of course, easy to achieve. Indeed, nearly any polychromatic beam emerging from a dichroic polarizer is likely to have a degree of polarization (and possibly even a polarization form) that varies with wavelength.

1.6. History of Understanding of Polarized Light. The history of man's understanding of polarized light has been touched on by Preston (P-30), Partington (P-10), and other authors. The early discoveries concerning double refraction are discussed in an anonymous work published in 1819 (A-10), and accounts of more recent developments are given by Archard (A-20), Johannsen (J-18), Thompson (T-5), Tutton (T-12), Twyman (T-13), and Wahlstrom (W-1). The development of the sheet-type polarizer has been described by Land and West (L-8) and by Land (L-13).

The following tabulation lists some of the principal advances:

1669 Erasmus Bartholinus, a Danish scientist, discovered double refraction (B-5).

1690 Christian Huyghens, a Dutch scientist, discovered polarization of light; he demonstrated the polarization with the aid of two calcite crystals arranged in series (H-42).

1757 Robert Hooke, an English physicist, tentatively suggested (perhaps before this year) that light vibrations are transverse (P-10).

1808 Etienne-Louise Malus, a French scientist, discovered polarization by

reflection. He happened to be looking through a calcite crystal at the light reflected obliquely from a window of the Luxembourg Palace, in Paris, and observed that the two images produced by the calcite were extinguished alternately as he rotated the crystal (G-12; P-10; M-12).

1811 D. F. J. Arago, a French scientist, discovered optical rotation (P-10).

1812 Jean B. Biot, a French physicist, was the first to appreciate that refracto-uniaxial crystals could be classified according to whether the extraordinary index was greater or less than the ordinary index (J-18).

1812 Arago invented the pile-of-plates polarizer (A-18).

1812 David Brewster, a Scottish physicist, discovered "Brewster's law" regarding polarization by reflection (Ref. B-48 makes it clear that the date of discovery was 1812, not 1811 as maintained by some authors).

1815 Biot discovered tourmaline's peculiar property, dichroism (B-25).

1816 Augustin Fresnel, a French physicist, found that two rays that are polarized at right angles cannot interfere. But he was unable to explain the matter until enlightened by Young (F-15).

1817 Thomas Young, an English physicist, was the first to prove that light vibrations are transverse, as had been suggested by Hooke in 1757 or before (Y-2; P-10).

1828 William Nicol, a Scottish physicist, invented the nicol prism (J-18; L-8).

1844 Wilhelm Haidinger, an Austrian mineralogist, discovered the "Haidinger's brush" phenomenon by means of which one may perceive directly that a broad, uniform beam of linearly polarized light is indeed polarized (H-6).

1845 Michael Faraday, an English physicist, discovered the Faraday effect.

1847 Haidinger discovered circular dichroism (P-10; H-7).

1852 William B. Herapath, an English physician, discovered a synthetic crystalline material that polarizes light of all wavelengths in the visual range (H-23, H-24; L-13; G-12).

1852 George G. Stokes, an English physicist, invented the four "Stokes parameters" for describing a beam of partially polarized light (S-29).

1875 John Kerr, a Scottish physicist, discovered the Kerr effect (K-9).

1887 Heinrich Hertz, a German physicist, produced Hertzian waves, giving much support to Maxwell's theory.

1892 Henri Poincaré, a French mathematician, invented the "Poincaré-sphere" method of representing a beam of polarized light (P-20; R-1).

1928 Edwin H. Land, then a 19-year-old student at Harvard College, invented the first successful sheet-type dichroic polarizer (L-4, L-8).

1933 Bernard F. Lyot, a young French scientist, invented the polarization-type, narrow-band filter that now bears his name (L-28).

1938 Edwin H. Land invented H-sheet, a dichroic sheet-type polarizer

that relies on a polymeric molecular type of absorber rather than a crystalline absorber.

1940 Robert Clark Jones, an American physicist, invented the Jones calculus for computing the changes produced in beams of light by polarizers and retarders (J-19).

1942 Francis Perrin, a French scientist, found how to put the four Stokes parameters and the sixteen Soleillet transformation constants into compatible form — a form involving matrix algebra (P-12).

1943 Hans Mueller, a professor of physics at Massachusetts Institute of Technology, invented in this year, or shortly before, a phenomenological approach to problems involving polarized and partially polarized light; the approach made extensive use of 4×4 matrices (M-26, M-28).

1943 Robert P. Blake, an American scientist, invented the HR-type polarizer, the first sheet-type polarizer suitable for use throughout an appreciable part of the infrared range (B-31; U.S. patent 2,494,686).

2

MODERN DESCRIPTION OF POLARIZED LIGHT

2.1. Introduction. We now turn to the modern methods of describing polarized light: the Poincaré sphere, the Stokes vector, the Jones vector, and the quantum-mechanical representation. We give particular attention to the Poincaré sphere and the Stokes vector because they provide direct insight into certain difficult problems and permit great simplification in various calculations of what happens to a beam when it encounters a succession of polarizers and retarders.

2.2. Poincaré Sphere. The Poincaré sphere, conceived by Henri Poincaré in about 1892 (P-20), provides a convenient way of representing polarized light and predicting how any given retarder will change the polarization form. The method is essentially one of mapping: each point on the sphere represents a different polarization form. The mapping may be carried out with the aid of a three-dimensional model, a two-dimensional projection, trigonometry, or analytic geometry. (The method is applicable only if the incident beam is completely polarized.)

In addition to its mapping function, the sphere leads to short-cut solutions of problems involving retarders, or combinations of retarders and ideal homogeneous polarizers. With the sphere in mind, one may often see at a glance the solution of a problem that would be difficult to solve by conventional methods. This is true, for example, of problems encountered by Pancharatnam (P-1, P-2) and Koester (K-12) in searching for a design of an achromatic, multilayer, retarder.

Despite its great usefulness, the Poincaré sphere has been given little

attention in the literature. The writer has been unable to find any substantial account of it in any textbook. However, brief descriptions have appeared in a number of technical journals; see Ramachandran and Ramaseshan (R-1, R-2), Jerrard (J-14), Koester (K-12), McMaster (M-2), Perrin (P-12), Skinner (S-17), Walker (W-3), and Wright (W-28).

Figure 2.1 indicates the significance of the various parts of the sphere. The upper and lower poles represent left- and right-circularly

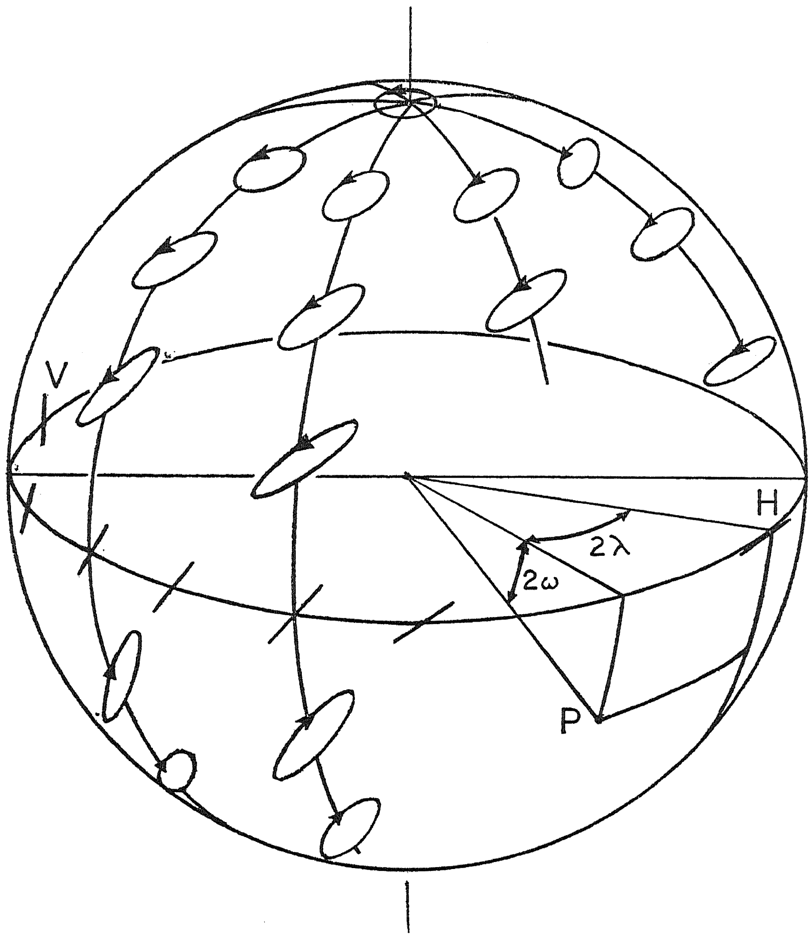


FIG. 2.1. The Poincaré sphere, showing the general significance of the different areas and the specification of the general point P in terms of the angles 2λ, measured clockwise from H, and 2ω, measured downward from the equator.

polarized light. Points on the equator indicate linear polarization. Other points indicate elliptical polarization. An arbitrarily chosen point H on the equator designates horizontal polarization, and the diametrically opposite point V designates vertical polarization. Beams whose polarization forms are orthogonal are represented by points at opposite ends of a diameter. Usually the radius of the sphere is assumed to be unity; sometimes it is regarded as proportional to the intensity of the beam.

A general point P on the surface of the (unit-radius) Poincaré sphere is specified in terms of the longitude (2λ) and the latitude (2ω), where $-180° \leq 2\lambda \leq 180°$ and $-90° \leq 2\omega \leq 90°$. (Note that the symbols λ and ω have quite different meanings from those implied in the previous chapter.) The longitude is positive when measured clockwise from point H; the latitude is positive when measured downward from the equator, that is, toward the pole representing right circularly polarized light. Thus the coordinates of the point P in Fig. 2.1 are positive.

The significance of the general point P is easily stated. It represents a completely polarized beam whose ellipse has azimuth λ, ellipticity $\tan|\omega|$, and a handedness that is left or right according to whether P lies in the upper or lower hemisphere. In summary, P represents an elliptically polarized beam whose sectional pattern is described thus:

$$\alpha = \lambda,$$
$$b/a = \tan|\omega|,$$

handedness: left or right for 2ω negative or positive respectively.

Obviously, each point on the sphere represents a different polarization form. Conversely, each polarization form is represented by a different point on the sphere.

Cartesian Coordinates. Alternatively, P may be specified by means of the right-handed cartesian coordinates X, Y, and Z indicated in Fig. 2.2. (These coordinates are not to be confused with those used for a different purpose in Chapter 1.) When the sphere has unit radius, the coordinates of P are:

$$X = \cos 2\omega \cos 2\lambda,$$
$$Y = \cos 2\omega \sin 2\lambda,$$
$$Z = \sin 2\omega.$$

When the light is horizontally polarized, $2\omega = 2\lambda = 0$, and the full expressions reduce to $X = 1$, $Y = 0$, $Z = 0$. When the light is

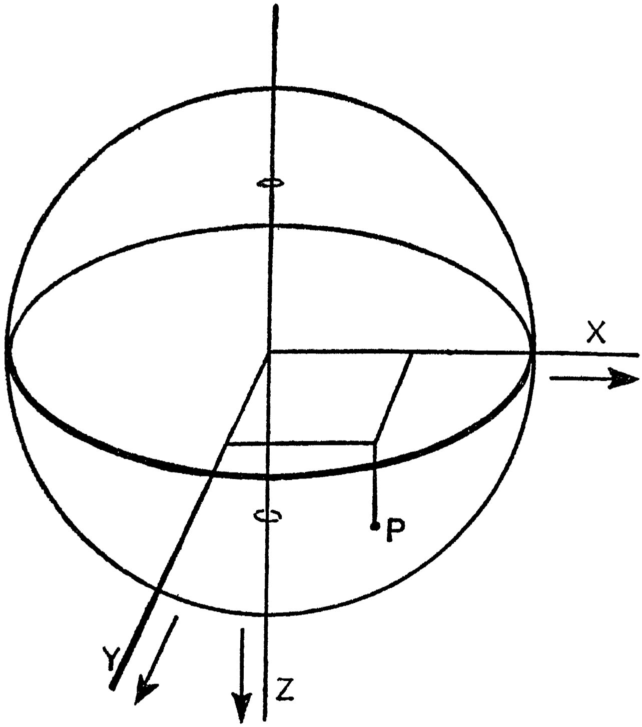


FIG. 2.2. Axes of the cartesian coordinate system used in specifying a point P on the unit-radius Poincaré sphere. Note that the positive Z-axis extends *downward.*

vertically polarized, $2\omega = 0$, $2\lambda = \pi$, and we find that $X = -1$, $Y = 0$, $Z = 0$. (We shall see in the next section that, in general, any completely polarized beam has X, Y, and Z values that are identical to the normalized Stokes parameters M, C, and S respectively. Thus the M, C, and S values shown in Table 2.1 are applicable here also.)

From the equations presented above, one sees that if the X, Y, and Z values of a beam are given, the azimuth, ellipticity, and handedness may be computed from the formulas

$$\alpha = \lambda = \frac{1}{2} \text{arc tan} \frac{Y}{X},$$

$$\frac{b}{a} = \tan |\omega| = \tan \left| \frac{1}{2} \text{arc sin } Z \right|,$$

handedness: left or right for Z negative or positive respectively.

The main use of the Poincaré sphere is in finding the effect of inserting a retarder in a polarized beam. The effect may be found merely by constructing an arc of a circle on the sphere. The method is described in detail in Chapter 7, and only a brief summary is needed here. The choice of point (on the sphere) to serve as center of the arc depends solely on the type of retarder (its fast eigenvector's azimuth, ellipticity, and handedness); the choice of arc length depends solely on the retardance. One end of the arc is defined by the point that describes the incident beam. The other end provides the answer: its location describes the emerging beam. Chapter 7, besides describing the procedure in detail, presents specific applications.

In general, the Poincaré-sphere representation is useful not merely in *solving* problems but also in *formulating* them — it clarifies one's thinking and talking. It is a kind of blackboard on which to indicate the starting point, the procedure, and the outcome, all with maximum economy of effort. The principal step, obviously, is drawing an arc or, to use a more common expression, *rotating the sphere*.

2.3. Stokes Vector. Though conceived in 1852 (S-29), the Stokes vector has received but scant attention in the literature of optics. Perhaps the best of the modern accounts is that by Walker (W-3). Other pertinent articles are those by Soleillet (S-24), Perrin (P-12), Billings and Land (B-22), Ramachandran and Ramaseshan (R-1), McMaster (M-2, M-4), Tinkham and Strandberg (T-7), Pancharatnam (P-4), Falkoff and McDonald (F-3). Various articles by R. Clark Jones, by H. Mueller, and by S. Chandrasekhar may be cited also; see bibliography. Van de Hulst's 1957 book (V-1) applies the Stokes vector to the problem of light scattering; Cohen's 1958 article (C-18) applies it to radio astronomy.

The Stokes vector consists of a set of four quantities (called the Stokes parameters) that describe the intensity and polarization of a beam of light. The beam may be polarized completely, partially, or not at all; it may be monochromatic or polychromatic. Thus the description, though very concise, is broadly applicable.

The four parameters have the dimensions of intensity; each corresponds not to an instantaneous intensity but to a time-averaged intensity, the average being taken over a period long enough to permit practical measurement. The vector, though consisting of four physically real parameters, is, of course, a *mathematical* vector; it exists in

a four-dimensional mathematical space, not in a three-dimensional physical space.

Following the practice adopted by Perrin and by Jones (P-12; J-19), we shall call the four quantities I, M, C, and S. (Stokes himself used A, B, C, D; Walker used I, Q, U, V). The four quantities comprise a column vector:

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix}.$$

The vector is often written horizontally, to save space; curly brackets are then used, as a reminder that the quantity is indeed a column vector:

$$\{I, M, C, S\}.$$

The importance of writing the quantities in an invariant order is made clear in Chapter 8, where the use of the vector in conjunction with 4×4 Mueller matrices is discussed.

The first parameter, I, is called the intensity. The parameters M, C, and S may be regarded as the "horizontal preference," "plus 45° preference," and "right circular preference" respectively. When a parameter has a negative value, the preference is for the orthogonal polarization form; thus if the parameter S has the value -0.5 the polarization form is more akin to left-circular polarization than to right-circular polarization.

Definitions. Although the parameters can be defined in terms of the electromagnetic theory, we prefer to define them operationally. This approach is perhaps more basic and reliable; also, it corresponds more closely to the historical development (S-29).

The operational approach makes use of a set of four filters. Any of several different sets could be used, but to make our account specific we shall assume a set of four filters F_1, F_2, F_3, F_4 that have the following properties: each has a transmittance of 0.5 for incident, unpolarized light; each is oriented so that its faces are vertical and are perpendicular to the beam; F_1 has the same effect on any incident beam, irrespective of the beam's polarization; in short, it is isotropic; F_2 is opaque to incident light that is polarized with its electric vibration direction at 90°, that is, vertical; F_3 is opaque to light polarized at $-45°$; F_4 is opaque to left-circularly polarized light.

Obviously the filters represent nonpolarizing, linearly (horizontally) polarizing, linearly (plus 45°) polarizing, and right-circularly polarizing filters respectively, as implied by Fig. 2.3.

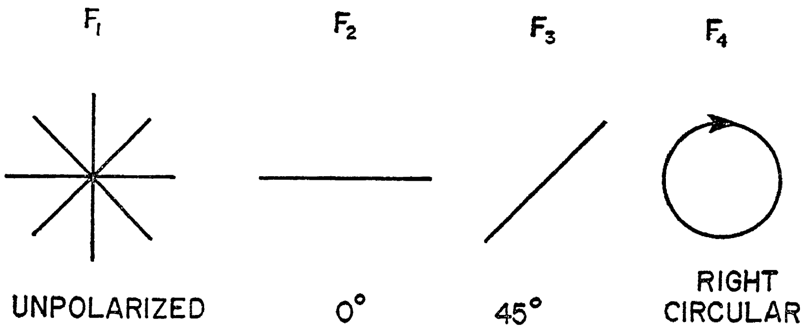


FIG. 2.3. Polarization forms produced by the four filters used in determining the four Stokes parameters of a beam.

In addition to the filters, a detector is employed. It is of polarization-independent type, and is calibrated in terms of intensity, or, more specifically, power. Its cross-sectional area must, of course, be at least as great as that of the beam in question.

The procedure is to place the detector squarely in the beam, interpose each of the four filters successively, note the four values indicated by the detector, and multiply each value by 1/0.5. From the resulting four values, which may be called V_1, V_2, V_3, and V_4 (and are not to be confused with the symbol that stands for degree of polarization!), we calculate the four parameters, thus:

$$\begin{aligned} I &= V_1, \\ M &= V_2 - V_1, \\ C &= V_3 - V_1, \\ S &= V_4 - V_1. \end{aligned}$$

The method of defining the parameters in terms of electromagnetic theory has been indicated by Walker (W-3). It is awkward in that one must assume that the light is sufficiently monochromatic that, at any time, a definable phase angle γ exists between the instantaneous scalar components a_x and a_y of the electric field, yet the light must be sufficiently *polychromatic* that the unpolarized state is not precluded. (The angle γ is defined so that, when $\pi > \gamma > 0$, the handedness of polarization is right; when $-\pi < \gamma < 0$ the handedness is left.)

The definitions are as follows (the angular brackets are employed to indicate that *time averages* are meant):

$$\begin{aligned} I &= \langle a_x^2 + a_y^2 \rangle, \\ M &= \langle a_x^2 - a_y^2 \rangle, \\ C &= \langle 2a_x a_y \cos \gamma \rangle, \\ S &= \langle 2a_x a_y \sin \gamma \rangle. \end{aligned}$$

As before, the four quantities have the dimensions of intensity. As before, M, C, and S may be positive, negative, or zero.

Normalized Parameters. Often we are interested only in relative values. We then divide all the parameters by the first one, to obtain the *normalized parameters.* Thus an original vector $\{4, 0, 2, 0\}$ becomes the normalized vector $\{1, 0, 0.5, 0\}$.

Typical Vectors. It is a simple exercise to compute, from the definitions based on electromagnetic theory, various vectors of principal interest. Consider, for example, unpolarized light. There is no time-averaged preference between a_x and a_y; consequently $\langle a_x^2 + a_y^2 \rangle$ reduces to $\langle 2a_x^2 \rangle$ and $\langle a_x^2 - a_y^2 \rangle$ reduces to zero. The multiplicative factor $\cos \gamma$ has a sign and magnitude that are independent of the sign and magnitude of the product $a_x a_y$; thus $\langle a_x a_y \cos \gamma \rangle$ reduces to zero. The same applies to $\langle a_x a_y \sin \gamma \rangle$. Consequently unpolarized light is described by an original vector $\{2a_x^2, 0, 0, 0\}$ and by the normalized vector $\{1, 0, 0, 0\}$.

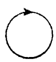
Similar reasoning shows that a horizontally polarized beam has the original vector $\{a_x^2, a_x^2, 0, 0\}$ and the normalized vector $\{1, 1, 0, 0\}$.

With little effort one may compute the Stokes vectors for many different forms of completely polarized light. Table 2.1 lists these vectors.

None of the parameters can be larger than the first (I). Each of the others lies in the range from $-I$ to $+I$. If the beam is entirely unpolarized, $M = C = S = 0$. If it is completely polarized, $(M^2 + C^2 + S^2)^{1/2} = I$. If the degree of polarization is V, it follows that $(M^2 + C^2 + S^2)^{1/2}/I = V$.

The parameter M is positive if the polarization form is more akin to a horizontal line than to a vertical line; it is negative if the preference is for a vertical line; it has the value zero when there is no preference between these, as for example when the polarization form is circular, or corresponds to an ellipse whose major axis is at $\pm 45°$. The parameter C is positive or negative depending on whether the

Table 2.1. Stokes vectors and Jones vectors of various forms of polarized light.

Polarization form					Normalized Stokes vector,	Jones vector	
Sectional pattern	α (deg)	b/a	A_y/A_x	γ (deg)	$\{I,\ M,\ C,\ S\}$	Standard normalized	Full
—	0	0	0	—	$\{1,\ 1,\ 0,\ 0\}$	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\0\end{bmatrix}$
|	90	0	∞	—	$\{1,\ -1,\ 0,\ 0\}$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}0\\A_ye^{i\epsilon_y}\end{bmatrix}$
/	45	0	1	0	$\{1,\ 0,\ 1,\ 0\}$	$\frac{\sqrt{2}}{2}\begin{bmatrix}1\\1\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\A_xe^{i\epsilon_x}\end{bmatrix}$
\	-45	0	1	± 180	$\{1,\ 0,\ -1,\ 0\}$	$\frac{\sqrt{2}}{2}\begin{bmatrix}1\\-1\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\-A_xe^{i\epsilon_x}\end{bmatrix}$
General	linear	0	Any positive number	0 or ± 180	$\{1,\ \cos 2\alpha,\ \sin 2\alpha,\ 0\}$	$\begin{bmatrix}\cos R\\\pm\sin R\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\\pm A_ye^{i\epsilon_x}\end{bmatrix}$
	—	1, R	1	90	$\{1,\ 0,\ 0,\ 1\}$	$\frac{\sqrt{2}}{2}\begin{bmatrix}-i\\1\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\A_xe^{i(\epsilon_x+\pi/2)}\end{bmatrix}$
	—	1, L	1	-90	$\{1,\ 0,\ 0,\ -1\}$	$\frac{\sqrt{2}}{2}\begin{bmatrix}i\\1\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\A_xe^{i(\epsilon_x-\pi/2)}\end{bmatrix}$
	0	$\frac{1}{2}$, R	$\frac{1}{2}$	90	$\{1,\ 0.6,\ 0,\ 0.8\}$	$\frac{2\sqrt{5}}{5}\begin{bmatrix}-i\\\frac{1}{2}\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\\frac{1}{2}A_xe^{i\left(\epsilon_x+\frac{\pi}{2}\right)}\end{bmatrix}$
	90	$\frac{1}{2}$, R	2	90	$\{1,\ -0.6,\ 0,\ 0.8\}$	$\frac{2\sqrt{5}}{5}\begin{bmatrix}\frac{-i}{2}\\1\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\2A_xe^{i\left(\epsilon_x+\frac{\pi}{2}\right)}\end{bmatrix}$
	22.5	0.318, R	0.518	45	$\left\{1,\ \sqrt{\frac{1}{3}},\ \sqrt{\frac{1}{3}},\ \sqrt{\frac{1}{3}}\right\}$	$0.325\begin{bmatrix}2.73\\1+i\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\0.518A_xe^{i\left(\epsilon_x+\frac{\pi}{4}\right)}\end{bmatrix}$
General elliptical					$\begin{bmatrix}1\\\cos 2\omega\cos 2\lambda\\\cos 2\omega\sin 2\lambda\\\sin 2\omega\end{bmatrix}$	$\begin{bmatrix}(\cos R)e^{-i\frac{\gamma}{2}}\\(\sin R)e^{i\frac{\gamma}{2}}\end{bmatrix}$	$\begin{bmatrix}A_xe^{i\epsilon_x}\\A_ye^{i\epsilon_y}\end{bmatrix}$
General elliptical, partially polarized			$\frac{1}{\langle a_x^2+a_y^2\rangle}$		$\begin{bmatrix}\langle a_x^2+a_y^2\rangle\\\langle a_x^2-a_y^2\rangle\\\langle 2a_xa_y\cos\gamma\rangle\\\langle 2a_xa_y\sin\gamma\rangle\end{bmatrix}$	None	None
Unpolarized					$\{1,\ 0,\ 0,\ 0\}$	None	None

polarization form is more akin to the $+45°$ direction than to the $-45°$ direction. The parameter S is positive for right-handed polarization forms, negative for left-handed forms, and zero for all linear forms.

Unit-intensity beams that are orthogonally polarized have Stokes vectors that differ only in the signs of the second, third, and fourth parameters; for example, the beams $\{1, 0.6, 0, 0.8\}$ and $\{1, -0.6, 0, -0.8\}$ are orthogonally polarized.

Principle of Optical Equivalence. After defining his vector, Stokes announced his "principle of optical equivalence" (S-29). According to this principle, beams that have the same Stokes vector are indistinguishable as regards intensity, degree of polarization, and polarization form.

(The principle is sometimes worded so as to imply that two beams having the same Stokes vector are indistinguishable. Such a statement is inaccurate; indeed, it is inaccurate even in cases where the two beams have, in addition, the same spectral energy distribution and the same geometry. For the fact is that the two beams will differ, in general, as regards phase, or fluctuations in phase; and by using an interferometric method such as that of Langsdorf and DuBridge (L-14) an investigator may be able to distinguish between the two beams.)

Types of Applications. The simplest application of the Stokes vector is to the combining of beams. When a beam 1 is combined with a beam 2 — and the two beams are *incoherent* — the properties of the combined beam c are found by adding the two vectors:

$$
\begin{aligned}
I_c &= I_1 + I_2, \\
M_c &= M_1 + M_2, \\
C_c &= C_1 + C_2, \\
S_c &= S_1 + S_2.
\end{aligned}
$$

Example: If the two initial beams have the vectors $\{3, 1, 1, -2\}$ and $\{4, 0, 0, 4\}$, the combined vector is $\{7, 1, 1, 2\}$.

Since the different areas of most light sources are incoherent with respect to one another, the Stokes vector is entirely applicable to the adding of beams originating at different areas of the source. (If the beams are coherent, or partially coherent, the procedure is not applicable, or is applicable only when certain precautions are taken. Pancharatnam, for example, has succeeded in using the Stokes vector in dealing with the interference of partially coherent beams; see

bibliography. An investigator who is concerned with coherent beams will usually prefer the Jones calculus.)

Perhaps the most important application of the Stokes vector is in connection with the Mueller calculus (Chapter 8), which affords a superb mathematical framework for computing the change produced in the intensity and polarization of a beam when it encounters a train of polarizers, retarders, and scatterers.

The Stokes vector of a completely polarized beam is closely related to the Poincaré-sphere representation. Consider the four quantities:

$$
\begin{aligned}
I &= a_x^2 + a_y^2, \\
M &= a_x^2 - a_y^2, \\
C &= 2a_x a_y \cos \gamma, \\
S &= 2a_x a_y \sin \gamma.
\end{aligned}
$$

As we have seen, $I^2 = M^2 + C^2 + S^2$. Consequently the first quantity may be regarded as the radius of a sphere and the other three may be regarded as the cartesian coordinates of a point on the sphere. The cartesian axis corresponding to the second quantity, $(a_x^2 - a_y^2)$, that is, M, is to be associated with horizontal polarization, since, for a given I, the quantity $(a_x^2 - a_y^2)$ is maximized when $a_y = 0$. Similarly the axes corresponding to the last two quantities are the Y- and Z-axes. In summary, the M, C, and S values of a completely polarized beam correspond to the X, Y, and Z values of the Poincaré-sphere representation of that beam.

2.4. Jones Vector. The Jones vector, introduced in 1941 by R. Clark Jones (J-19), describes a polarized beam with the maximum algebraic brevity, and is eminently suited to the solving of problems involving beams whose phase relations are important. The present section defines the Jones vector, lists its commonest forms, and relates it to the more familiar ways of describing polarized light. Also, it shows how the vector is used in problems involving the combining of polarized beams of any polarization form and any phase relationship. (The use of the vector in problems involving polarizers and retarders is treated in Chapter 8.)

The Jones vector is a two-element column vector which describes a beam's polarization form and amplitude components at some given position along the beam. If the beam is traveling along the Z-axis, the vector has the general form:

$$\begin{bmatrix} m \\ n \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} E_x \\ E_y \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} A_x e^{i(\epsilon_x + 2\pi\nu t)} \\ A_y e^{i(\epsilon_y + 2\pi\nu t)} \end{bmatrix},$$

where E_x and E_y are the scalar components (of the instantaneous electric vector) along the X- and Y-axes; A_x is the maximum value of E_x, and A_y is the maximum value of E_y. The quantity ϵ_x is the phase of the component E_x at time $t = 0$ and at the given location; ϵ_y is the phase of the component E_y. (As in Chapter 1, the letter R may be used to represent arc tan A_y/A_x.) In general, each element of the column vector is a complex quantity.

We may convert the vector to the following equivalent form:

$$e^{i2\pi\nu t} \begin{bmatrix} A_x e^{i\epsilon_x} \\ A_y e^{i\epsilon_y} \end{bmatrix}.$$

Since the magnitude of any quantity of the form e^{iM} is unity, the magnitude of $e^{i2\pi\nu t}$ is unity. Thus this latter quantity may be dropped entirely in those problems in which details of variation with time are not of interest. Most problems are of this type, and accordingly the Jones vector is often written in the following form, called the *full Jones vector:*

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ A_y e^{i\epsilon_y} \end{bmatrix}.$$

Obviously, if it turns out that the time variation is of interest, the factor $e^{i2\pi\nu t}$ can be restored at any desired stage of the calculation.

In certain cases the full vector can be simplified further. Consider horizontally polarized light: here $A_y = 0$, so the vector reduces to

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ 0 \end{bmatrix}.$$

For light that is linearly polarized at 45°, the two quantities A_x and A_y are equal, the quantities ϵ_x and ϵ_y are equal, and the vector reduces to

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ A_x e^{i\epsilon_x} \end{bmatrix} \quad \text{or} \quad A_x e^{i\epsilon_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

For right-circularly polarized light, $A_x = A_y$ and $\gamma \equiv \epsilon_y - \epsilon_x = \frac{1}{2}\pi$. Thus the vector is

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ A_x e^{i(\epsilon_x + \frac{1}{2}\pi)} \end{bmatrix}.$$

Table 2.1 lists the full Jones vectors for various forms of polarization.

Intensity of the Beam. The intensity of the beam is proportional to the sum of the squares of the magnitudes of the individual elements. If the units of intensity or amplitude are chosen so that the proportionality constant is unity, the relation becomes

$$I = A_x^2 + A_y^2.$$

Standard Normalized Jones Vector. Any full vector may be converted to standard normalized form by multiplying it by whatever scalar (ordinarily a complex scalar) reduces the intensity to unity and also reduces the vector to simplest form. The process is called normalizing. Examples of nonnormalized and normalized vectors are as follows:

Nonnormalized Form	*Standard Normalized Form*
$\begin{bmatrix} 3e^{i\epsilon_x} \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\frac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\frac{1}{2}\begin{bmatrix} 1+i \\ 1-i \end{bmatrix}$	$\frac{\sqrt{2}}{2}\begin{bmatrix} i \\ 1 \end{bmatrix}$

Table 2.1 lists the standard normalized vectors for various forms of polarized light.

If a normalized Jones vector is denoted by $\begin{bmatrix} m \\ n \end{bmatrix}$, the vector of an orthogonally polarized beam is $\begin{bmatrix} -n^* \\ m^* \end{bmatrix}$, where the asterisk denotes the complex conjugate. Thus, for example, the following two forms are orthogonal:

$$\frac{\sqrt{2}}{2}\begin{bmatrix} -i \\ 1 \end{bmatrix} \quad \text{and} \quad \frac{\sqrt{2}}{2}\begin{bmatrix} -1 \\ i \end{bmatrix}.$$

Interpretation of the Vector. The polarization form implied by a given Jones vector may be expressed in conventional terms with the aid of the following "tabulation of inferences":

Status of E_x and E_y	*Inference as to Polarization Form*
$E_x \neq 0, \quad E_y = 0$	Linear; $\alpha = 0°$
$E_x = 0, \quad E_y \neq 0$	Linear; $\alpha = 90°$
$E_x \neq 0, \quad E_y \neq 0, \quad E_x = E_y$	Linear; $\alpha = 45°$
$E_x \neq 0, \quad E_y \neq 0, \quad E_x = -E_y$	Linear; $\alpha = -45°$
$E_y/E_x = i$	Right circular
$E_y/E_x = -i$	Left circular
General case	Elliptical; see below

When a vector not falling in one of the simple cases is encountered, a three-step procedure is followed. Step 1 consists in converting the vector to the form of the full Jones vector:

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ A_y e^{i\epsilon_y} \end{bmatrix}.$$

Thus if an investigator encounters the vector

$$\begin{bmatrix} 0.3i \\ 0.5e^{i(-0.2)} \end{bmatrix},$$

he alters the first element by replacing i by its equivalent $e^{i\pi/2}$, and obtains:

$$\begin{bmatrix} 0.3e^{i\pi/2} \\ 0.5e^{i(-0.2)} \end{bmatrix}.$$

Step 2 consists in computing the angles R and γ, defined in the previous chapter as $|\text{arc tan } (A_y/A_x)|$ and $(\epsilon_y - \epsilon_x)$. In the present example,

$$R = |\text{arc tan } (0.5/0.3)| = 59.0°,$$
$$\gamma = (-0.2 - \tfrac{1}{2}\pi) = -1.77 \text{ rad} \quad \text{or} \quad -101°.$$

Step 3 consists in employing the following rules:

If $\sin \gamma > 0$, the ellipse is right-handed; if $\sin \gamma < 0$, the ellipse is left-handed;

The azimuth α of the major axis is given by

$$\alpha = \tfrac{1}{2} \text{ arc tan } [(\tan 2R) (\cos \gamma)];$$

The ellipticity b/a is given by

$$b/a = \tan \beta,$$

where $\beta = \frac{1}{2} \text{ arc sin } (\sin 2R\, |\sin \gamma|)$.

Applying these rules to the present example, one finds that:

$\sin\gamma < 0$, and consequently the ellipse is left-handed;

$$\begin{aligned}\alpha &= \tfrac{1}{2}\arctan[\tan 118^\circ \cos(-101^\circ)]\\ &= \tfrac{1}{2}\arctan 0.37 = 10^\circ,\\ \beta &= \tfrac{1}{2}\arcsin[\sin 118^\circ|\sin(-101^\circ)|]\\ &= \tfrac{1}{2}\arcsin 0.86 = 30^\circ,\\ b/a &= \tan 30^\circ = 0.58.\end{aligned}$$

An investigator who is familiar with the Jones vector can easily compute the result of combining two coherent beams of completely polarized light. He writes down the two full vectors appropriate to the given polarization forms, given intensities, and given phases. He then adds the vectors.

Consider, for example, the result of adding two linearly polarized beams, one of which is polarized horizontally and the other vertically. Let each beam have an intensity of 3 units, and let the vertically polarized beam be 90° ahead in phase. The two full vectors are:

$$\begin{bmatrix}\sqrt{3}e^{i\epsilon_x}\\ 0\end{bmatrix} \text{ and } \begin{bmatrix}0\\ \sqrt{3}e^{i(\epsilon_x+90^\circ)}\end{bmatrix}.$$

The sum is

$$\begin{bmatrix}\sqrt{3}e^{i\epsilon_x}\\ \sqrt{3}e^{i(\epsilon_x+90^\circ)}\end{bmatrix} = \sqrt{3}e^{i(\epsilon_x+90^\circ)}\begin{bmatrix}e^{-i(90^\circ)}\\ 1\end{bmatrix} = \sqrt{3}e^{i(\epsilon_x+90^\circ)}\begin{bmatrix}-i\\ 1\end{bmatrix}.$$

Referring to Table 2.1 one sees that this last expression represents a beam that is right-circularly polarized and has an intensity of $2(\sqrt{3})^2$ or 6.

The most important applications of the Jones vector are in connection with the Jones calculus (Chapter 8), which is the most succinct of the algebraic methods of computing the outcome of passing a completely polarized beam through a series of ideal retarders and polarizers. Other applications are discussed by Ramachandran and Ramaseshan (R-2).

2.5. Quantum-mechanical Description. The great majority of problems involving polarizers and polarized light are solved without reference to quantum mechanics. However, the relation between the quantum-mechanical specification of light and the more conventional specifications (presented in the foregoing sections) is of interest. The relation has been discussed by Jauch and Rohrlich (J-6), Fano (F-5, F-6), and McMaster (M-5).

According to the quantum-mechanical viewpoint, a given beam of completely polarized, monochromatic light consists of photons that are identical. Each is in the identical state (a pure state), and has the same wave function. This is true not only of a linearly polarized beam, but also of a circularly or elliptically polarized beam; the wave function that applies is different, of course, for each polarization type. In general, no one polarization type is more basic, or simpler, than any other; however, if one is dealing mainly with spin operators, right- and left-circularly polarized photons are specified with maximum simplicity.

Even an *unpolarized* (monochromatic) beam consists, according to the quantum-mechanical viewpoint, of identical photons, and each of these has the same wave function. However, the wave function represents a mixed state, that is, an incoherent combination, described by an expression of the form

$$|a\psi_1| + |b\psi_2|,$$

whereas the specification of the general, pure state (representing the general case of elliptical polarization and constituting a coherent combination) has the form

$$|a\psi_1 + b\psi_2|.$$

The statement that all the photons of the unpolarized beam are identical may at first seem surprising, but is in accord with the important experimental fact that, a priori, our expectation concerning the behavior of any one photon (from the given unpolarized beam) is the same as the expectation concerning any other photon of this beam. It is in accord, in short, with the fact that an unpolarized beam contains no information as to its history, that is, as to whether it was created by the incoherent combination of two orthogonally linearly polarized forms, two orthogonal circular forms, two orthogonal elliptical forms, or in some more complicated way.

The Stokes vector, also, takes no account of the history of the given beam but depends solely on expectation probabilities. Accordingly, its relation to the quantum-mechanical description of light is a close one. Indeed, the Stokes vector serves as a kind of bridge between the classical theory of light and the quantum mechanical theory; see, for example, McMaster's monographs M-2 and M-5.

2.6. Thermodynamics and Entropy. A completely polarized, nearly monochromatic ray of given intensity can be assigned a temperature

and also a value of entropy flux, as explained by Planck (P-17, P-18). If the beam is only partially polarized, two temperatures are involved, that is, a different temperature must be assigned to the dominant component and the inferior component, defined in Sec. 1.5. Likewise two values of entropy flux are involved. If the beam is entirely unpolarized, the two temperatures are identical, and the two values of entropy flux are identical. (If it is completely polarized, we may still say that two temperatures exist, and that one of the temperatures is zero; likewise we may say that two entropy fluxes exist, and that one of these is zero.)

A 1-watt beam of 500-mμ light that is completely polarized has a temperature that is higher than that of either component of a 1-watt, 500-mμ beam that is unpolarized (and has the same width and angular spread); the entropy flux of the polarized beam is less than the sum of the entropy fluxes of the two components of the unpolarized beam. Of course, the temperature and the entropy flux of a completely polarized beam are independent of the polarization form concerned.

For additional information, see Born (B-41), Jones (J-30), Ore (O-3), and Rosen (R-12).

The relation of polarized light and general relativity has been discussed by Mariot (M-14).

3 POLARIZERS: CLASSES AND PERFORMANCE PARAMETERS

3.1. Introduction. There are several methods of producing polarized light directly, that is, without recourse to a polarizer. They involve the Stark effect, the Zeeman effect, grazing emergence, biemissivity, bifluorescence, the Cherenkov effect, and various other effects. Appendix 1 describes the methods in some detail. Few, if any, of the methods are of practical importance in the production of polarized light.

The usual, practical method of producing polarized light is to use a conventional lamp of some sort to produce light (unpolarized light) and a polarizer to polarize it by transmitting one component and eliminating the orthogonal component.

This chapter classifies polarizers and defines their performance parameters. The definitions must be formulated so as to take account of the two different roles that a polarizer may play: the production of polarization (polarizing role) and the detection of polarization (analyzing role).

3.2. Definition of a Polarizer. A polarizer is a purely optical device that, when supplied only with unpolarized light, can produce a beam that is appreciably polarized. Depending on whether the polarization type is linear, circular, or elliptical, the polarizer is called a *linear*, *circular*, or *elliptical* polarizer.

Some polarizers consist of a single uniform layer, and are called homogeneous. Others employ several different layers and are called inhomogeneous, or multilayer. An inhomogeneous polarizer may ap-

pear to belong to two different classes, depending on which face the light emerges from. The commercially produced CP-HN-38 circular polarizer consists of two layers: (*a*) a linearly polarizing layer, and (*b*) a linearly retarding layer (having a certain retardance and orientation). If the light is incident on *a* and emerges from *b*, it emerges circularly polarized. If it is incident on *b* and emerges from *a*, it emerges linearly polarized. In addition to this linear-circular combination, other two-layer, dual-function polarizers can be constructed, for example, a linear-elliptical combination. Using three layers one can make, for example, a right-circular–left-circular combination (ambidextrous polarizer), or a linear 0°–linear 45° combination.

A variable-axis-direction polarizer is a side-by-side arrangement, or mosaic, of elements each of which is a linear polarizer and has a *different* orientation.

A *spectrally selective* polarizer is one whose properties change appreciably with wavelength. If the selectivity occurs in the visual range of the spectrum, the polarizer is called a *colored* polarizer. Depending on whether it is designed to control the hue or the saturation of the beam, it is called a *variable-hue* polarizer or a *variable-saturation* polarizer. Polarizers that are not spectrally selective are called *achromatic* or *neutral.*

A *sheet-type* polarizer is one whose thickness is negligible compared to its width.

The adjective *spathic* has been applied by Jones (J-28) to a polarizer that conserves the cross-sectional area, solid angle, and wavelength distribution of the beam.

A polarizer that has been cemented between protective covers of glass or plastic is called a *laminated* polarizer.

Most polarizers have some depolarizing tendencies. Scattering is one frequent cause of this tendency; many polarizers exhibit a detectable amount of scattering. Other causes are oblique reflections, edge effects, and internal strains. A depolarizing polarizer is one that, though capable of increasing the degree of polarization of a beam that is initially unpolarized, will unavoidably *reduce* the degree of polarization of an incident beam that is already 100-percent polarized. The depolarizing tendency of most high-quality polarizers is so small as to be negligible in nearly all types of applications. (Many of the laws governing polarizers are valid only if the depolarizing tendency is negligible.)

A polarizing beam splitter is, of course, a polarizer. It divides the beam into two components, and these ordinarily exhibit orthogonal polarization.

A retarder is a polarization-form *converter*, but is not itself a polarizer. (Chapter 7 discusses retarders in detail.)

3.3. Optical Mechanisms Employed. The polarizer's task is to divide the incident beam into two components that are orthogonal as to polarization form, transmit one component, and absorb or divert the other. Four mechanisms for accomplishing this are available: dichroism, birefringence, reflection, and scattering.

Dichroism is perhaps the most important mechanism. The great majority of currently produced polarizers are of dichroic type. A dichroic polarizer preferentially transmits one polarization form and absorbs the orthogonal form. Since the latter form is disposed of *in situ*, no auxiliary diaphragms or absorbing surfaces are needed. J-sheet, the first of the well-known, sheet-type, dichroic polarizers, relies on oriented, dichroic microcrystals; it is known as a *microcrystalline* polarizer. H-sheet and various other modern types rely on dichroic molecules — long slender molecules that have been brought into alignment; such polarizers are called *molecular*. Dichroic polarizers are discussed in detail in Chapter 4.

A typical birefringence polarizer consists of two prisms of calcite cemented together. The combination divides the incident beam into two components, transmits one, and totally internally reflects the other toward a blackened surface. The best-known design, invented by William Nicol in 1828 (N-5), is still in use in certain special applications, although other designs have proved superior in many applications. These polarizers are described in Chapter 5.

Reflection plays the central role in several naturally occurring polarization processes, and in one well-known type of infrared polarizer. Whenever unpolarized light, proceeding in air or vacuum, is reflected obliquely from the surface of a glass plate or other smooth-faced dielectric object, the reflected beam is polarized to some extent. By using favorable obliquity, and by employing a number of plates in series, a designer can construct a reflection polarizer that performs well throughout a wide spectral range.

Scattering is another mechanism whereby polarization can be accomplished. The polarization of light from the blue sky is a conse-

quence of scattering. X-rays and gamma-rays have been polarized successfully by means of a scattering polarizer. Reflection and scattering polarizers are discussed in Chapter 6.

3.4. Principal Performance Parameters of a Linear Polarizer. We deal here with the performance parameters of a linear polarizer, concerning ourselves not with the *intensive* properties of the material comprising the polarizer, but with the *extensive* properties of the polarizer as a whole.

If the linear polarizer is of homogeneous type, either face may be chosen as the face for the light to emerge from. In practice, few polarizers are perfectly homogeneous; consequently the two faces are not equivalent. For example, one face may exhibit a small amount of scattering, or a small amount of retardance due to strain in the cover glass or cement. The name *prime face* is used to designate the face that must be employed as emergence face if the resulting beam is to have the greatest degree of polarization. We shall ordinarily assume that the polarizer is oriented so that its prime face is the emergence face if the polarizer is to be used to create polarization, and is the incidence face if the polarizer is to be used as an analyzer.

One important parameter of a polarizer is its *polarizance* P, defined (B-28) as the degree of polarization produced by the polarizer when the incident beam is unpolarized. (Degree of polarization V is, as explained in Chapter 1, a property of a beam; polarizance P is a property of a polarizer. If the beam incident on a polarizer is partially polarized already, the degree of polarization of the emerging beam will differ from the polarizance of the polarizer.)

Perhaps the most useful parameters of a linear polarizer are the two principal *transmittances:* the major principal transmittance k_1 and the minor principal transmittance k_2. The parameter k_1 is defined (L-8, W-22) as the ratio of transmitted intensity to incident intensity when the incident beam is linearly polarized in that vibration azimuth that maximizes the transmittance. The ratio obtained when the transmittance is minimized is called k_2. The k_1 value of a high-quality polarizer may be nearly as great as unity, and k_2 may be nearly as small as zero. The ratio k_1/k_2, represented by R_t, is called the principal transmittance ratio; the R_t value of a high-quality polarizer may be as large as 10^5 for wavelengths near the center of the polarizer's useful range (B-8). Often, an investigator prefers to deal with k_1 and k_2 values that have

been corrected for reflection losses that occur at the surfaces; the loss amounts typically to about 4 percent per surface.

The expression *intensity transmittance* is sometimes used. In working with the Jones calculus, an investigator often deals with $\sqrt{k_1}$ and $\sqrt{k_2}$, quantities that have been assigned the symbols p_1 and p_2 and are called the major and minor *amplitude transmittances.*

Instead of dealing with the principal transmittances, an investigator may deal with the *principal densities.* The minor and major principal densities are called d_1 and d_2, and are defined as $\log_{10}(1/k_1)$ and $\log_{10}(1/k_2)$ respectively. The ratio d_2/d_1 is called the principal density ratio, R_d. If the polarizer is of dichroic type, the term *dichroic ratio* may be used in place of density ratio; and the name *dichroitance* may be used for the density difference $D_d \equiv (d_2 - d_1)$.

The principal transmittance ratio R_t is a parameter of value to the user of a polarizer, since in many applications it corresponds to the ratio of the intensities of the wanted and unwanted components of the emerging beam. Persons attempting to discover new and better dichromophores prefer to deal with the principal density ratio R_d, since this is more nearly independent of the areal concentration of the dichromophore and hence can be used as a figure of merit of the dichromophore. Values of R_d of 30 to 50 are usually deemed excellent, and values below 5 or 10 are usually considered unsatisfactory.

The average of the principal transmittances is called the *total transmittance;* the symbol k_t is used. Thus $k_t = \frac{1}{2}(k_1 + k_2)$. Obviously, k_t is the ratio of transmitted intensity to incident intensity when the incident beam is unpolarized. If the incident beam is polarized, the ratio of transmitted intensity to incident intensity (called the *actual* transmittance) will differ from k_t. Whereas the quantity k_t is a constant of the polarizer, the actual transmittance is a variable; it depends not only on the polarizer but also on the polarization of the incident beam.

The quantity G_d, defined as $k_2/(k_1 + k_2)$ and called *polarization defect* of the polarizer (S-11), has been used as a measure of the shortcoming in its performance.

The *transmission axis* of a linear polarizer is defined with respect to a beam of linearly polarized light that is incident normally on the prime face of the polarizer. The direction which the electric vibration of the beam must have in order that the actual transmittance be

maximized is the transmission axis. Obviously, the axis is a direction, not one actual line; for convenience, it is usually pictured as a straight line through the center of the polarizer, as in Fig. 3.1. In describing the transmission-axis direction, one usually assumes that the polarizer is situated in a beam traveling horizontally in the positive Z-direction; the X-axis is horizontal, and the Y-axis vertical; the source is situated at the origin, and the observer is situated far out along the Z-axis, looking toward the polarizer and the source beyond. The azimuth of the transmission axis is indicated by the angle θ, measured counterclockwise from the X-axis.

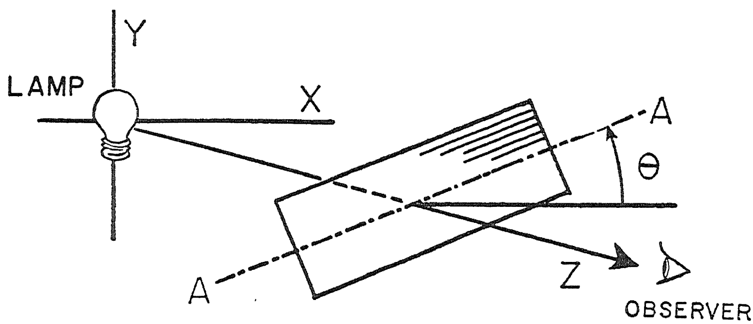


FIG. 3.1. Azimuth θ of the transmission axis AA of a linear polarizer situated in a beam traveling horizontally along the positive Z-axis.

The properties defined above are applicable when the incident light is monochromatic, or roughly monochromatic. Often the source emits a broad band of wavelengths, and the detector is responsive to a broad band. In such cases no simple procedure is applicable categorically; one must deal with each wavelength separately, and then find how to combine the outcomes — if, indeed, a combination is meaningful. When the wavelength is changed, nearly every property of the polarizer may change. For example, the choice of prime face may change; the transmission axis may change; the principal transmittances may change. A majority of the commercially produced linear polarizers remain surprisingly constant — in most respects — over a spectral range of at least one-half octave and, in some instances, one octave or more.

In some instances, a polarizer's property may be weighted in a standard way, when a wide spectral range is involved. This is true, for example, of the total transmittance k_t. If the source corresponds

to I.C.I. Illuminant C (daylight) and the detector is the average, photopic, human eye, the *visual-range total luminous transmittance* k_v may prove to be a useful quantity. It is defined as

$$k_v = \frac{\int F_a(\lambda) F_b(\lambda) k_t(\lambda)\, d\lambda}{\int F_a(\lambda) F_b(\lambda)\, d\lambda},$$

where λ is the wavelength in vacuum, F_a is a weighting function taking account of the spectral radiance curve of the lamp, and F_b is a weighting function taking account of the spectral responsivity of the photopic eye (as specified, for example, in Ref. O-2). The integration is to be carried out over the entire visual range of the spectrum.

Another "broad-band" parameter is k_{vx}, the visual-range transmittance of a crossed pair of identical polarizers. It too may be expressed as a ratio of integrals; the integrals are the same as those presented above except that the quantity $k_t(\lambda)$ must be replaced by the product $k_1(\lambda)k_2(\lambda)$.

3.5. Performance Parameters of a Pair of Identical Polarizers. As polarizers are often tested in pairs and used in pairs, parameters applicable to the pair are needed. The *transmittance of the parallel pair*, called H_0, is the ratio of transmitted intensity to incident intensity, when (*a*) the two identical polarizers are placed together, prime face to prime face, in an unpolarized (monochromatic) beam, and (*b*) the azimuth of the second polarizer is adjusted relative to that of the first polarizer so as to maximize the transmittance. The *transmittance of the crossed pair*, H_{90}, is the ratio found when one polarizer is turned so as to minimize the transmittance.

The symbols G_0 and G_{90} are used for the density of the parallel pair and the density of the crossed pair; G_0 is defined as $\log_{10}(1/H_0)$, and G_{90} is defined as $\log_{10}(1/H_{90})$.

When the two identical polarizers are of ideal, homogeneous, nondepolarizing type, the relations between k_1, k_2, H_0, and H_{90} are easily stated. As shown by West and Jones (W-22):

$$\begin{aligned}
H_0 &= \tfrac{1}{2}(k_1^2 + k_2^2),\\
H_{90} &= k_1 k_2,\\
k_1 &= \tfrac{1}{2}\sqrt{2}[(H_0 + H_{90})^{1/2} + (H_0 - H_{90})^{1/2}],\\
k_2 &= \tfrac{1}{2}\sqrt{2}[(H_0 + H_{9c})^{1/2} - (H_0 - H_{90})^{1/2}].
\end{aligned}$$

In many cases k_2 is so small compared to k_1 (and H_{90} so small compared to H_0) that the equations may be greatly simplified without appreciable loss of accuracy.

Law of Malus. When a beam of unpolarized monochromatic light is incident on two ideal linear polarizers arranged in series, the transmittance depends, of course, on the angle between the transmission axes of the two polarizers, that is, on θ, the *angle of crossing*. If each polarizer is of homogeneous type and has k_1 and k_2 values of 1.0 and 0.0 (and thus has H_0 and H_{90} values of 0.5 and 0.0), the relation between the transmittance of the pair and the angle of crossing is as follows:

$$H(\theta) = 0.5(\cos^2 \theta).$$

This law, due to Malus, is perhaps the single most useful law of polarizer technology. (If a beam of 100-percent linearly polarized light encounters an ideal polarizer at azimuth θ relative to the vibration direction of the incident beam, the actual transmittance is 1.0 $\cos^2 \theta$.)

If unpolarized light strikes an identical pair of polarizers for which H_0 and H_{90} do not have the ideal values of 0.5 and 0.0, the following equation applies:

$$H(\theta) = H_{90} + (H_0 - H_{90}) \cos^2 \theta.$$

Of course, if the polarizers have depolarizing tendencies, or are of multilayer type, a more complicated equation may apply; see Chapter 8.

Usually one assumes that the light striking a polarizer is incident perpendicularly. If it is incident obliquely, various *obliquity effects* arise. Baxter (B-9) has shown that the obliquity effects of the HN-22 dichroic polarizer are small, provided the obliquity is less than about 30°. For a typical birefringence polarizer, the obliquity effects tend to be more serious, as shown by Groosmuller (G-13); see also Hasse (H-4).

3.6. Circular and Elliptical Polarizers. Circular polarizers are either of right-handed or left-handed type, that is, they produce either right-circularly or left-circularly polarized light.

As explained in Chapter 7, circular polarizers are commonly made by laminating together two special layers (a linear polarizer and a linear 90° retarder). Accordingly, specification of the *prime face* is usually

essential. Obviously, the face that is nearer to the retarder than to the linear polarizer is the prime face; this face must be used as the emergence face if the emerging light is to exhibit circular polarization.

Although a homogeneous circular polarizer has, in principle, no preferred (transverse) direction, an *inhomogeneous* circular polarizer is likely to exhibit some preference. Preference will result if the retarder has an inappropriate retardance or is cemented to the linear polarizer at an inappropriate azimuth. Consequently it is often desirable to indicate, as a supplementary piece of information, the transmission axis of the linearly polarizing layer or the fast axis of the retarder (Sec. 5.2).

With respect to a circular polarizer, k_1 is defined as the transmittance found when a circularly polarized beam incident on the prime face has the handedness that maximizes the transmittance; k_2 is the transmittance found when the handedness of the incident beam is such as to minimize the transmittance. As before, $k_t = \frac{1}{2}(k_1 + k_2)$.

When a circular polarizer is placed in series with another circular polarizer having similar k_1 and k_2 values and with the prime faces adjacent, the resulting transmittance parameter may be called the *transmittance of the similar pair* (H_{sim}) or the *transmittance of the orthogonal pair* (H_{orth}), depending on whether the two polarizers have similar or opposite handedness.

The general polarizer is a device that produces, from an unpolarized beam, an elliptically polarized beam. The polarizer may consist of one layer or several layers; in the latter case the prime face must be identified. The general homogeneous polarizer is customarily specified in terms of the polarization form of the light that emerges from it when the incident beam is unpolarized. The polarization form may be specified in terms of the ellipticity, the handedness, and the azimuth of the major semiaxis, as in Chapter 1, or in terms of the Stokes vector, Poincaré-sphere representation, or Jones vector, as in Chapter 2. For the general polarizer, as for the linear or circular polarizer, quantities k_1 and k_2 may be defined straightforwardly, and suitable pair parameters also.

"Round-trip" parameters. Circular polarizers are often used to suppress light that is reflected perpendicularly from a smooth, polarization-conserving surface S situated close behind the circular polarizer (Sec. 9.8). In essence, the purpose of the polarizer is to prevent the light from making a "round trip" through the polarizer to the surface S

and back through the polarizer. The fraction of light that accomplishes the round trip may be called the *round-trip nominal total transmittance* (k_{ntrt}). The quantity ($1 - k_{ntrt}$) may be called the *nominal suppression efficiency, E_{ns}*. The ratio k_t/k_{ntrt} may be called the "nominal factor of improvement resulting from suppressing specular reflection"; it is a sort of factor of merit, taking into account the (wanted) one-way transmission of light and (unwanted) round-trip transmission of light.

3.7. Eigenvector of a Polarizer. The preceding sections have classified polarizers according to the polarization form of the emerging beam. Little or no attention has been paid to the relation of the polarizer to the polarization (if any) of the *incident* beam. Such attention is sometimes needed. Consider, for example, a typical, two-layer circular polarizer, consisting of a linearly polarizing layer and a retarding layer (Chapter 7). If the incident beam is already linearly polarized, the choice of azimuth of its vibration has an enormous effect on the intensity of the transmitted beam. (This would not be true, of course, if the circular polarizer were of homogeneous type: the intensity of the transmitted beam would then be independent of the azimuth of the incident beam.)

To characterize a polarizer in such a manner as to take into account its backward-looking properties as well as its forward-looking properties, one may specify its *eigenvectors* (assuming the polarizer has no depolarizing tendency). An eigenvector is a polarization form that satisfies the following condition: if an incident, 100-percent polarized beam has this form, the emerging beam also has this form. Consider, for example, a polarizer that has the k_1 and k_2 values 0.9 and 0.1. If the transmission axis is horizontal, the two eigenvectors of the polarizer are *horizontal linear polarization* and *vertical linear polarization;* these may be called the major eigenvector and minor eigenvector respectively. If the polarizer's k_2 value is zero, there is only one eigenvector, namely the major eigenvector. A piece of clear isotropic glass has an infinite number of eigenvectors.

Bodies that exhibit dichroism and also circular birefringence may have nonorthogonal eigenvectors, as shown by Pancharatnam (P-5).

An eigenvector may, of course, be specified in terms of the sectional pattern, Poincaré sphere, Stokes vector, or Jones vector (Chapter 2).

Multilayer polarizers often have eigenvectors that neither maximize nor minimize the actual transmittance. Consider, for example, a two-

layer polarizer consisting of linearly polarizing layers A and B, the transmission axes of which are at 0° and 45° (Fig. 3.2). Let us assume that each layer's k_1 and k_2 values are 1.0 and 0.0, and that each layer is free of retardance and exhibits no depolarizing tendency. Then the eigenvector of the combination is linear polarization at 45°, parallel to

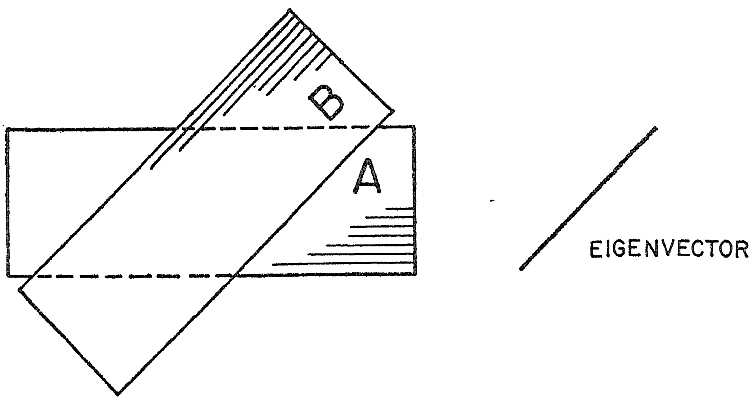


FIG. 3.2. Eigenvector of a polarizer consisting of linearly polarizing layers A and B the axes of which are at 45° to one another. The k_1 and k_2 values of each layer are 1.0 and 0.0. The incident beam is normal to the plane of the paper, and encounters layer A first.

the transmission axis of the second layer (layer B). The actual transmittance with respect to the eigenvector of the combination is 0.25, whereas a greater value (0.50) would be achieved if the vibration direction of the incident beam were at 0°.

3.8. Mueller Matrices and Jones Matrices. The most versatile method of describing a polarizer is by means of its *Mueller matrix*, a 4×4 array of 16 real elements. Such a matrix can describe any polarizer, whether or not it exhibits retardance and scattering in addition to polarization, and whether it consists of one layer or many layers. If the polarizer is free of scattering, the 2×2 *Jones matrix* may be used.

Chapter 8 describes these matrices in detail and shows how they are used. They are eminently useful in problems involving a train of several polarizers, a train of several retarders, or a train containing polarizers and retarders. Certain problems that formerly appeared almost hopelessly complicated can be solved easily using these new tools.

4 DICHROIC POLARIZERS

4.1. Introduction. The great majority of commercially produced polarizers are *dichroic* polarizers: they exhibit *dichroism*, the property of absorbing light to different extents depending on the polarization form of the incident beam.

Persons concerned with color analysis and spectrophotometric analysis use the term *dichroic* in three other ways. They apply it to (1) a dye solution that exhibits a large change in color when the concentration of the solution is changed by a large factor, (2) a color filter that has two transmission bands in very different portions of the visual spectrum and hence may exhibit a large change in color when the spectral energy distribution of the illumination is changed, and (3) an interference filter that appears differently colored when viewed by transmitted or reflected light. (A term that is perhaps better suited to those uses is *dichromatic*.) The application of the term *dichroic* to absorboanisotropic bodies goes back over a hundred years, and is said to have been suggested by the tendency of various crystals of this class to exhibit color changes when viewed through a slowly rotating polarizer.

This chapter deals not with the ultimate causes of dichroism, but only with those manifestations of it that play important parts in the design and manufacture of linear polarizers. Dichroic *circular* polarizers are discussed in Chapter 7 and the cause of dichroism in Chapter 10; see also a review by West (W-21).

4.2. Dichroism and Absorboanisotropy. The general homogeneous body is not isotropic. Its absorption properties, refraction properties, x-ray properties, and so forth, differ with direction. Accordingly terms such as absorboanisotropy, refractoanisotropy, and the like may be applicable.

This chapter deals with absorboanisotropic bodies, that is, bodies that exhibit dichroism. A plate cut from such a body will divide an obliquely incident ray into two polarized components and will absorb these to different extents. To simplify the discussion we shall assume that the body is *linearly dichroic*, and thus produces components that are *linearly* polarized. (We shall assume also that the absorption occurring in a path one wavelength long is very small, and that the body does not scatter the light.)

It should be emphasized that the extent of absorption of a component depends on its vibration direction. Propagation direction is not the basic consideration: beams having the same propagation direction may be absorbed to different extents if their vibration directions differ; but if the vibration directions are the same, the absorptions will be the same even when the propagation directions differ.

In dealing with a body that absorbs radiation, one usually specifies the absorbance or absorbancy. Absorbance is defined as $\log_{10} (I_o/I)$, where I_o is the intensity of the incident beam and I is the intensity of the emerging beam. Absorbancy is simply absorbance per unit path length, $(1/L) \log_{10} (I_o/I)$.

The terms and symbols to be used in what follows differ from those used by various other authors. Many authors deal only with refraction, only with absorption, or only with x-ray properties. The present book deals with all these properties; consequently the adoption of a more versatile set of symbols is a clear necessity.

Consider an experiment in which an investigator directs a beam of linearly polarized light at the most general type of dichroic body, and progressively alters the directions of propagation and of vibration of the incident beam. If, for each choice of conditions, he determines the absorbancy, he will find that different conditions lead, ordinarily, to different values. The maximum value that may be found is called the major principal absorbancy coefficient a_w, and the minimum value is called the minor principal absorbancy coefficient a_u. The wave normals of the various beams governed by a_w are coplanar; in other words, there is one and only one direction that is perpendicular to all of

these. This direction is called the major principal absorption axis W. Likewise the wave normals of the various beams governed by a_u are perpendicular to a single direction, called the minor principal absorption axis U. These two axes are mutually perpendicular; the axis that is normal to both of them is the intermediate principal absorption axis V; associated with it is the intermediate principal absorbancy coefficient a_v.

We are now in a position to define the *absorption indicatrix*, which is the most succinct statement of the absorption properties of the absorbing body. It is the ellipsoid whose semiaxes are parallel to W, V, and U and have lengths proportional to a_w, a_v, and a_u.

The absorption indicatrix serves as a three-dimensional model from which the two (nonprincipal) absorbancy coefficients associated with any given wave-normal direction may be found. The procedure, as suggested in Fig. 4.1, is to cut the indicatrix with a plane that passes

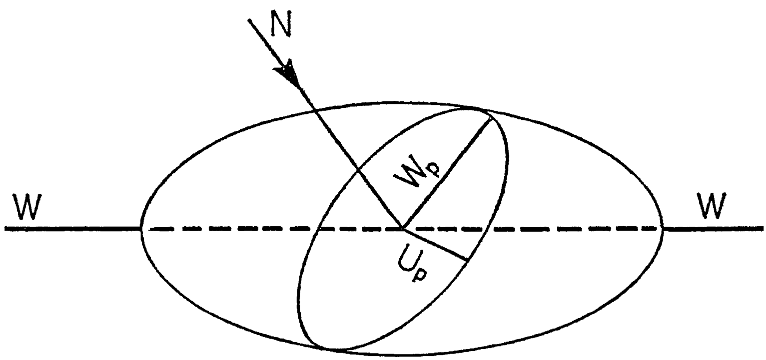


FIG. 4.1. Method of finding the two (nonprincipal) absorbancy coefficients associated with the given wave-normal direction N and the given absorption indicatrix. The coefficients are proportional to the lengths of the semiaxes W_p and U_p of the ellipse normal to N.

through the center of the indicatrix and is normal to the given wave normal; the plane intersects the indicatrix in an ellipse, the major and minor semiaxes of which are called W_p and U_p. The lengths of these semiaxes correspond to the two absorbancy coefficients in question.

Let us now consider some general plane that passes through the center of the indicatrix. One can always orient the plane so that its intersection with the (ellipsoidal) indicatrix is a circle. The normal to the plane is then called a *monochroic axis* A_m, and the angle between

this axis and the W-axis is the monochroic semiangle M. For the general indicatrix (whose semiaxes are all different in length) there are two monochroic axes, and the angle $2M$ between them is called the *monochroic angle* (Fig. 4.2).

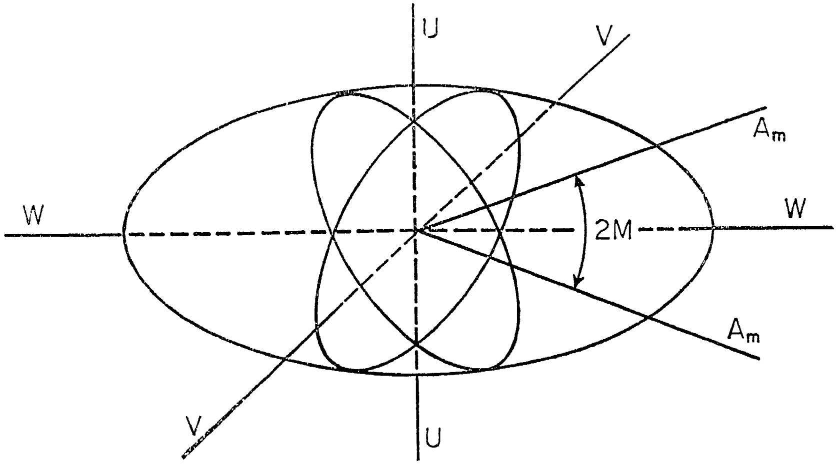


FIG. 4.2. Monochroic axes A_m and the monochroic angle $2M$ of an absorboanisotropic body. In this instance, $2M < 90°$; accordingly the indicatrix is said to be prolate, or positive.

When there are two monochroic axes, the body is called absorbobiaxial, or *pleochroic;* if the two monochroic axes coincide, the body is called absorbouniaxial. An indicatrix is called prolate (or positive) if $2M < 90°$ and oblate (or negative) if $2M > 90°$.

The term *dichroism*, when used quantitatively, means the difference between the principal absorbancy coefficients. The quantity $(a_w - a_v)$ may be called the first dichroism, D_{wv}; $(a_v - a_u)$ is the second dichroism, D_{vu}; $(a_w - a_u)$ is the over-all dichroism, D_{wu}. The last quantity is the most important one, and accordingly the adjective "overall" is often omitted.

4.3. Planodichroism. When a beam is incident normally on a flat polarizer, the two polarized components within the polarizer have, of course, the same wave-normal direction (the direction perpendicular to the entrance face of the polarizer). Consequently we are concerned with but a single cross section of the absorption indicatrix, namely, the cross section that is parallel to the entrance face. The (qualitative)

term *planoabsorption* may be given to the absorption processes concerned.

In the general case the cross section is elliptical. The principal axes of the ellipse may be called the major and minor planoabsorption axes W_p and U_p, and the lengths of the semiaxes represent the *major* and *minor planoabsorbancy coefficients* a_{wp} and a_{up}. The difference $(a_{wp} - a_{up})$, called D_p, is the *planodichroism*. The ratio $R_p = a_{wp}/a_{up}$ is the *planodichroic ratio*. Obviously, the quantities a_{wp}, and a_{up}, D_p, and R_p are *intensive* parameters: they are independent of the body's dimensions.

Let us now consider a polarizer that consists of a thin plate cut from a large homogeneous specimen of absorboanisotropic material. In general, the planoaxes of the plate differ from the principal axes of the material comprising the plate; likewise, the planoabsorbancy coefficients of the plate differ from the principal absorbancy coefficients of the material. (However, if the body is uniaxial and thus the indicatrix is an ellipsoid of revolution, one planoaxis will necessarily be the same as one of the principal axes, and one planoabsorbancy coefficient will necessarily be the same as one of the principal absorbancy coefficients.)

The relation between the intensive and extensive properties of the plate is simple. The product of thickness and planodichroism is the (extensive property) *dichroitance*. The product of thickness and a_{wp} is the major principal density, d_2. The product of thickness and a_{up} is the minor principal density, d_1.

4.4. Planodichroism of Bodies Containing Oriented Microscopic Units. A dichroic polarizer may, of course, consist of a single crystal, for instance, a crystal of tourmaline. Indeed, it was while studying tourmaline that Biot, in 1815, discovered dichroism (B-25, G-12, P-10, L-8). However, a polarizer consisting of a single crystal of tourmaline has many shortcomings, such as small aperture, low dichroic ratio, short spectral range, and high cost.

In the late 1920's Edwin Herbert Land found that the long-established limitations on polarizer aperture could be overcome by relying not on one single crystal, but on a statistical array of small, dichroic particles (long, thin microcrystals) brought into alignment. To minimize light scattering, he employed crystals whose thickness was much less than a wavelength of light. He aligned the particles with respect

to their longest axis; and, since he had taken the precaution of selecting a type of crystal such that the longest axis coincided with the major principal absorption axis, aligning the crystals with respect to shape was tantamount to aligning them with respect to the long axis of the absorption indicatrix. Accordingly the resulting array acted like a single polarizing unit having almost unlimited length and breadth. Such an array is called a *microcrystalline polarizer*. Later, Land and his colleagues developed the *molecular polarizer*, which employed aligned, long, thin dichroic molecules. These various kinds of polarizers are discussed in detail in later sections.

The individual, microscopic, dichroic entity is often called a *dichromophore*. Sometimes this name is reserved for one particular part of the absorbing entity, the part that is responsible for the dichroism.

The dichroic properties of the individual microcrystal can usually be specified straightforwardly by means of the terminology of the previous section. However, the same may not be true of a dichromophore that consists of a molecule. It is usually impractical to study a single molecule; yet if many molecules are taken together, an interesting difficulty arises: the molecules make contact with one another and may influence one another's properties. How, then, can the investigator distinguish between the absorption properties of the individual molecule, of a closely packed group of molecules, and of such a group after the individuals have been brought more or less into alignment? Stated differently: what part of the performance is a measure of the molecule *per se*, what part is a measure of the proximity effects, and what part is a measure of the degree of alignment? Practical answers to such questions are difficult to find.

The commonest method of orienting millions of needle-shaped dichromophores (embedded, for example, in a plastic sheet of nitrocellulose or polyvinyl alcohol or other rubber-elastic material) is to stretch the sheet unidirectionally. Needles that originally have random directions tend to be turned so that all are approximately parallel to the stretch axis (work axis); the resulting orientation is called *linear parallelism*.

If, instead of unidirectionally stretching the sheet, one unidirectionally compresses it, the needles tend to become parallel to a plane, namely, the plane normal to the compression axis, but show no preferred direction within this plane. This orientation is called *uniplanar parallelism*.

In most instances, the dichromophores are needle-shaped and have a prolate (positive) absorption indicatrix; also, the long axis is in most instances parallel to the major principal absorption axis. When a sheet containing such needles is stretched to a considerable extent, the major principal absorption axes of the needles become approximately parallel to the stretch direction; consequently the resulting polarizer has an absorption axis parallel to this direction. A polarizer exhibiting this identity of absorption axis and stretch axis is said to exhibit *parallel* planoabsorption. It is possible to make a polarizer having an absorption axis that is perpendicular to the stretch axis; such a polarizer is said to exhibit *perpendicular* planoabsorption. (Some authors use the terms positive and negative in place of parallel and perpendicular, and thus invite confusion with a separate subject, namely the sign of the absorption indicatrix.)

It is apparent that, in a microcrystalline polarizer, *two* absorption indicatrices are involved, one for the individual crystal, and another for the composite material, that is, for the dispersion of millions of submicroscopic dichroic crystals in an isotropic medium. Suppose, for example, that the individual crystal is needle-shaped, the indicatrix is prolate, and the major axis of the indicatrix is parallel to the long axis of the needle; suppose also that the crystals are brought into linear parallelism; then the indicatrix of the composite material will prove to be prolate. Suppose, on the other hand, that the crystals are brought into uniplanar parallelism; then the indicatrix of the composite material is oblate. Figure 4.3 illustrates these two geometries, which may be called Class P_l and Class P_u respectively. (Note that the symbols l and u here stand for *l*inear parallelism and *u*niplanar parallelism; the symbols P and O stand for *p*rolate and *o*blate shapes of the indicatrix of the individual crystal.)

Consider, now, the case in which the individual needle has an oblate indicatrix. If the long axis of the needle is parallel to the shortest axis of the indicatrix, the indicatrix of the composite material will be oblate when the needles exhibit linear parallelism (Class O_l geometry) and prolate when they exhibit uniplanar parallelism (Class O_u geometry). These same principles apply, of course, to polarizers containing dichroic molecules instead of dichroic crystals.

Jones (J-22, J-23) has shown that when the geometry is of Class O_u, the density ratio R_d of the polarizer cannot exceed 2.0 even when the individual crystal is needle-shaped and its indicatrix also is needle-

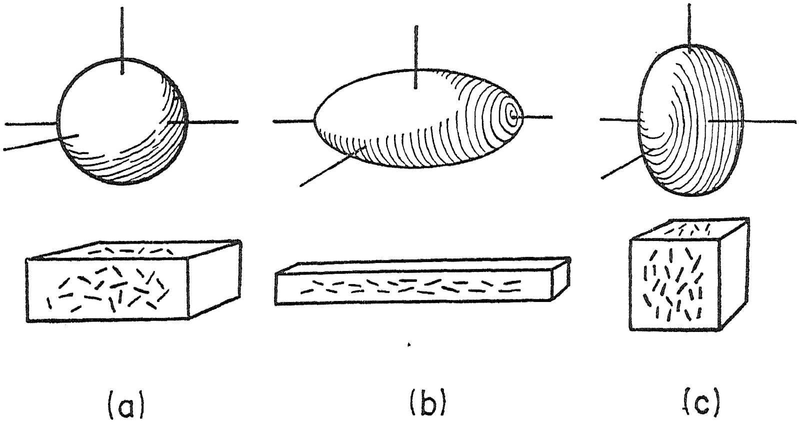


FIG. 4.3. Two types of parallelism that may be produced in a composite material containing needle-shaped dichroic crystals that individually have a prolate indicatrix (with major principal absorption axis parallel to the longest axis of the needle). (*a*) Strip prior to stretching; needles have random directions; indicatrix of composite material is a sphere. (*b*) Strip has been stretched unidirectionally; needles have a high degree of linear parallelism; indicatrix of composite material is prolate. (*c*) Strip has been compressed unidirectionally; needles have a high degree of uniplanar parallelism; indicatrix of composite material is oblate.

shaped. However, for a Class O_l polarizer, and likewise a Class P_l or P_u polarizer, no upper limit can be established a priori.

4.5. Means of Aligning Dichromophore Molecules. The more successful types of dichromophore molecules are those that have (1) strong absorption and large dichroism throughout a large range of wavelength, (2) an absorption band that is due to a structure having a pronounced electric dipole moment within the molecule, (3) a long thin shape, and (4) an absorption axis that is parallel to the long axis.

The dichromophore molecules may be aligned by (*a*) dispersing them in a homogeneous isotropic material, then unidirectionally stretching the specimen (extruding or shearing may be used instead of stretching); (*b*) attaching them chemically within a homogeneous material that already has a high degree of parallelism; (*c*) attaching them to the *surface* of a sheet that already has a preferred direction explicit on its surface (the preferred direction may be a consequence of a unidirectional rubbing operation); (*d*) forming, that is, creating, them within a homogeneous material that already has a high degree of parallelism; (*e*) miscellaneous means involving, for example, mag-

netic or electrostatic fields. Detailed accounts of many different ways of aligning the dichromophores are presented in U.S. patents by Land and his colleagues. The majority of these patents are listed in the bibliography.

Most of the commercially produced sheet-type polarizers are made by processes involving the stretching of large plastic sheets. A commonly used plastic is the polymer *polyvinyl alcohol.* Stretching may be accomplished by a machine employing several sets of rollers running at different speeds; warming the sheet facilitates stretching, reduces the force required, and lessens the danger of breaking the sheet. The center line of the stretching (*machine axis*) is parallel, or nearly parallel, to the absorption axis. Near the edge of the sheet, the actual direction of stretching (*strain axis*, or *work axis*) may differ slightly from the machine axis. Sometimes the terms X-, Y-, and Z-axes are employed; they refer to the normal to the sheet, the transverse direction, and the machine axis, respectively.

As a measure of the extent of stretching, one may record the ratio of final length to initial length (the stretch ratio); however, this quantity takes no account of the decrease in width that accompanies the increase in length. A better measure is the *axial ratio:* one inscribes a circle on the sheet before starting the stretching operation; after the operation is complete, one measures the major and minor axes of the resulting (elliptical) figure and takes the ratio of the former to the latter.

Jones (J-23, J-24) has computed the maximum value of the density ratio R_d that can be achieved by stretching a sheet (containing dichroic microcrystals) to a specified axial ratio. One of his findings is that the density ratio of the composite material can never exceed the dichroic ratio of the individual crystal, and in any event cannot exceed about 1.3 times the axial ratio. No exact computations have been made for the molecular polarizer.

4.6. H-*Sheet.* The most widely produced type of sheet polarizer is the type known as H-sheet (manufactured by the Polaroid Corporation, Cambridge, Massachusetts). This molecular-type polarizer was invented in 1938, or shortly before, by E. H. Land. H-sheet may be defined generally as a transparent linear polymeric material that (1) consists, in large part, of polymeric molecules that have a preferred direction and (2) has been stained with a material that causes the

sheet to exhibit dichroism. The H-sheet usually produced commercially consists mainly of a sheet of polyvinyl alcohol that has been unidirectionally stretched; also it contains iodine that has been imbibed from a liquid solution, called H-ink, that is rich in iodine. In this book the term H-sheet usually means this particular variety.

Manufacture. The manufacture of H-sheet is in accord with methods described in U.S. patent 2,454,515 and in various other patents and technical articles by Land, West, and others (L-8, L-13, B-36). Typically, a thin sheet of polyvinyl alcohol (PVA) is heated, stretched, and promptly laminated to a *support sheet* of cellulose acetate butyrate. The PVA face of the assembly is then brought into contact with H-ink, and imbibes a small amount of iodine from the ink. In some instances an isotropic dye that absorbs ultraviolet light but is transparent to visible light is included in the polarizer, to absorb radiation that does not contribute to visibility. Finally, the sheet is dried, rolled up, and held ready for cutting into pieces of such sizes as are desired. To achieve a sheet exhibiting a high value of k_1 and a very low value of k_2, the machine operators must control the principal chemical and physical conditions with great care; to make a polarizer of indifferent quality is easy, but to make a large sheet that will exhibit very high polarizance and will be uniformly satisfactory in other respects is far from easy.

Pieces cut from the roll of *supported H-sheet* are laminated between protective cover plates of plastic or glass. A typical finished polarizer is disk-shaped and is perhaps 1 to 3 inches in diameter. Disks 18 inches in diameter can be made. Rectangular pieces may be 18 inches wide and of almost unlimited length.

The amount of iodine that lodges in the polarizer depends on many variables, including the concentration of iodine in the ink, the temperature of the ink, and the duration of the inking process. Three grades of H-sheet, differing in the amount of iodine present in unit area of the polarizer, are produced in quantity: HN-22, HN-32, and HN-38. (The letter H designates the general type of polarizer, N stands for *neutral color*, and the accompanying number is (approximately) the percentage total luminous transmittance k_t of a single polarizer.)

Table 4.1 lists the principal transmittance values of HN-22, HN-32, and HN-38 at representative wavelengths in the visual range. Figure 4.4 presents the data in graphic form. Table 4.2 lists the values of

Table 4.1. Principal transmittances k_1 and k_2 of HN polarizers.[a]

Wavelength ($m\mu$)	HN-22		HN-32		HN-38	
	k_1	k_2	k_1	k_2	k_1	k_2
375	0.11	0.000,005	0.33	0.001	0.54	0.02
	(.02)	(.000,001)	(.05)	(.000,1)	(.07)	(.003)
400	.21	.000,01	.47	.003	.67	.04
	(.12)	(.000,01)	(.24)	(.002)	(.34)	(.02)
450	.45	.000,003	.68	.000,5	.81	.02
	(.45)	(.000,003)	(.68)	(.000,5)	(.81)	(.02)
500	.55	.000,002	.75	.000,05	.86	005
550	.48	.000,002	.70	.000,02	.82	.000,7
600	.43	.000,002	.67	.000,02	.79	.000,3
650	.47	.000,002	.70	.000,02	.82	.000,3
700	.59	.000,003	.77	.000,03	.86	.000,7
750	.69	.000,01	.84	.000,2	.90	.004

[a] Data are from Refs. S-15 and P-26. The data are valid in applications in which no reflection losses occur. Although these values are believed to be typical, allowance must be made for variations in the manufacturing process. The values in parentheses refer to polarizers containing ultraviolet absorber.

parallel and crossed transmittances of an identical pair. The k_{vx} values are approximately 0.000,005, 0.000,05, and 0.000,5 (S-15). The principal transmittance ratio R_t of HN-22 may be as large as 10^5 at the most favorable wavelengths (B-8). The transmission-axis direction is perpendicular to the stretch direction.

It is apparent that the three grades of H-sheet provide high polarizance P and large major principal transmittance k_1 throughout a wide range of wavelengths. However, the performance of HN-38 is noticeably imperfect near the short-wavelength end of the visual range: the k_1 value falls here, and the k_2 value rises; thus R_t becomes much smaller. A pair of crossed HN-38 polarizers transmits a noticeable amount of light of wavelength below 470 $m\mu$; hence the expression *blue leak.* The near-470-$m\mu$ crossed transmittance is less in the HN-32 polarizer, and much less in HN-22. Consequently HN-22 is used in applications where very low leakage (high extinction) is necessary; HN-38 is used where very large k_1 is desired and a larger leakage is tolerable; HN-32 represents a compromise that is well suited to many applications.

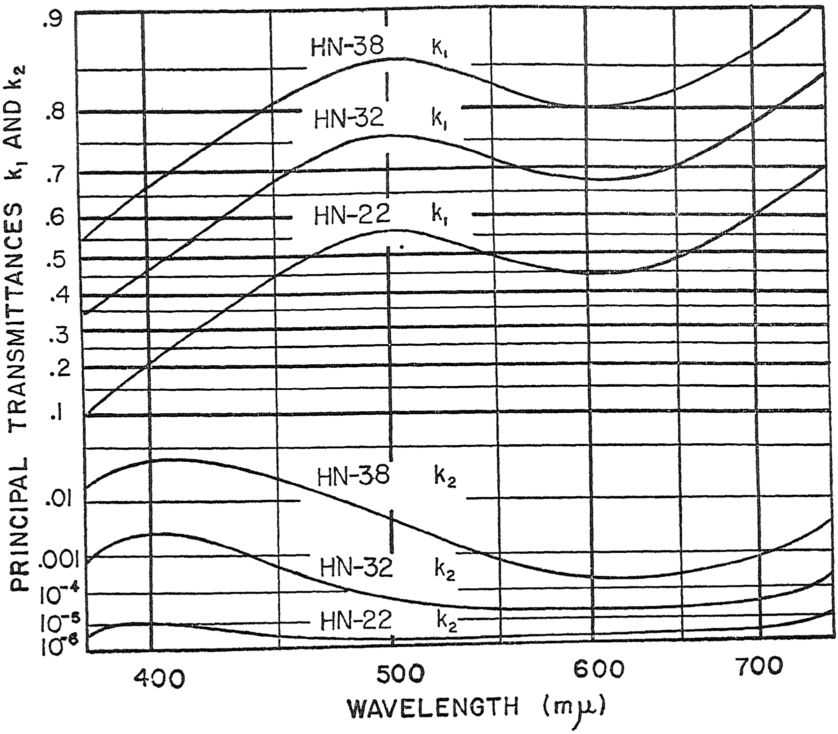


FIG. 4.4. Curves of k_1 and k_2 for three grades of HN polarizers: HN-22, HN-32, and HN-38. The vertical scale is linear in log log$(1/k)$.

Useful performance in the near infrared is achieved (out to at least 900 mμ) by employing several aligned H-type polarizers in series; the density ratio remains high here, and if several layers are used in series satisfactorily high polarizance results. The performance in the near ultraviolet tends to be disappointing, but if the absorption by the supporting sheet and cover plates can be obviated useful performance down to about 300 mμ may be achieved (B-3).

The chemical form of the iodine present in H-sheet has been investigated by Land and West (L-8, W-18). It appears to consist of long-chain polymer molecules; in a given chain the successive iodine atoms are about 3.10 angstrom units apart. This separation, West points out, is characteristic neither of iodine atoms that are joined by single bonds nor of the successive groups in PVA (whose unit-cell length is 2.52 A). Thus one infers that a PVA-iodine complex of some

Table 4.2. Parallel-pair transmittance H_0 and crossed-pair transmittance H_{90} of HN polarizers.[a]

Wavelength (mμ)	HN-22		HN-32		HN-38	
	H_0	H_{90}	H_0	H_{90}	H_0	H_{90}
375	0.006	0.000,000,5	0.05	0.000,3	0.15	0.01
	(.000,2)	(.000,000,02)	(.001)	(.000,05)	(.002)	(.000,2)
400	.02	.000,002	.11	.002	.22	.03
	(.007)	(.000,001)	(.03)	(.000,5)	(.06)	(.01)
450	.10	.000,001	.23	.000,3	.33	.02
	(.10)	(.000,001)	(.23)	(.000,3)	(.33)	(.02)
500	.15	.000,001	.28	.000,04	.37	.004
550	.12	.000,001	.25	.000,01	.34	.000,6
600	.09	.000,001	.22	.000,01	.31	.000,2
650	.11	.000,001	.25	.000,01	.34	.000,2
700	.17	.000,002	.30	.000,02	.37	.000,6
750	.24	.000,007	.35	.000,2	.41	.004

[a] Data are from Refs. S-15 and P-26. The data are valid in applications in which no reflection losses occur. Although these values are believed to be typical, allowance must be made for variations in the manufacturing process. The values in parentheses refer to polarizers containing ultraviolet absorber.

sort is created, and that it is this complex that determines the 3.10-A spacing. The typical length of the PVA chains and polymeric iodine chains is not known, although rough estimates may be made by means of a formula proposed by Kuhn (K-22).

Polyvinyl alcohol has been studied at length by Krimm *et al.* (K-20), Clarke and Blout (C-17), Blout and Karplus (B-32), Cohen *et al.* (C-19), and others; see also D-24, E-10, H-1. Several methods of preparation are available, and lead to products having slightly different properties. PVA film may be produced from a PVA solution by means of casting; the cast film has a refraction indicatrix (Sec. 5.2) that, for light in the visual range, is oblate, the shortest axis being perpendicular to the film; when the film is stretched unidirectionally, the indicatrix becomes prolate, the long-axis direction being parallel to the stretch direction. The over-all coefficient of birefringence may be as great as 0.034 (Land and West, L-8). In certain portions of the infrared region, PVA itself exhibits absorption and, moreover, dichroism; such properties, explored by Krimm *et al.* (K-20) and by

Tadokoro *et al.* (T-1), add to one's understanding of the structure of PVA and contribute also to the understanding of the mechanism of iodine uptake.

H-sheet itself exhibits birefringence, and this is, of course, related to the dichroism. Mielenz and Jones (M-19) have investigated the relation. Blake *et al.* (B-31) have measured the birefringence in the near infrared.

H-sheet exhibits excellent stability throughout a reasonably wide range of operating temperatures, but tends to deteriorate if exposed to high temperature (175°F) or to a combination of high temperature and high humidity. The deterioration is much slower if the H-sheet has been laminated between cover plates of plastic or glass.

Varieties of H-sheet involving other plastics and other dichromophores have been prepared. Sheets of polyvinyl butyral were used successfully by Land in 1938 (U.S. patent 2,454,515). Other kinds of ink have been used. Polarizers employing PVA have been made in a number of institutions; the Russian scientist Godina (G-8) has discussed some of the properties of such polarizers. See also references K-3 and C-28 regarding polarizers made by Käsemann in Germany, and patent 2,246,087 by Bailey and Brubaker. Reference should be made also to the "Bernotar" polarizer produced in recent years in Jena, Germany. The dichroic ratio of an individual crystal of iodine has been measured by Bovis (B-47).

4.7. K-Sheet. K-sheet, invented by E. H. Land and H. G. Rogers in 1939 or shortly before, is perhaps the second most important type of sheet polarizer. It ranks close behind H-sheet in commercial importance and surpasses it in certain respects, such as resistance to effects of high temperature and high humidity.

The manufacture of K-sheet starts with a sheet of polyvinyl alcohol (PVA). The main step involves not adding any atoms to the PVA, but taking atoms away. The PVA sheet is heated in the presence of a catalyst (for example, HCl) and as a result $2n$ hydrogen atoms and n oxygen atoms are given off. The process is called catalytic dehydration, and has been described in various patents and articles (patents 2,173,304 and 2,306,108; L-8, L-13, B-36, and J-9). Stretching operations are used to align the long-chain molecules (see, for example, Rogers' patent 2,255,940). The resulting material is laminated to a

sheet of supporting material such as cellulose acetate butyrate and is then laminated between cover plates of plastic or glass.

Table 4.3 lists the transmittance values of KN-36, which is the usual grade of K-sheet. Figure 4.5 presents similar data in graphic

Table 4.3. Principal transmittances k_1 and k_2, parallel-pair transmittance H_0, and crossed-pair transmittance H_{90} of KN-36 polarizers.[a]

Wavelength (mμ)	k_1	k_2	H_0	H_{90}
375	0.42	0.002	0.09	0.001
	(.03)	(.001)	(.000,5)	(.000,03)
400	.51	.001	.13	.000,5
	(.31)	(.001)	(.05)	(.000,3)
450	.65	.000,3	.21	.000,2
	(.65)	(.000,3)	(.21)	(.000,2)
500	.71	.000,05	.25	.000,04
550	.74	.000,04	.27	.000,03
600	.79	.000,03	.31	.000,02
650	.83	.000,08	.34	.000,07
700	.88	.02	.39	.02
750	.92	.57	.59	.53

[a] Data are from Refs. S-15 and P-26. The data are valid in applications in which no reflection losses occur. Although these values are believed to be typical, allowance must be made for variations in the manufacturing process. The values in parentheses refer to polarizers containing ultraviolet absorber.

form. K-sheet performs satisfactorily throughout most of the visual range. The performance is less satisfactory near the long-wavelength end of this range: when two crossed K polarizers are held in front of a white-light source, the (faint) transmitted light has a reddish hue.

The transmission axis is perpendicular to the stretch direction.

The dichromophore in K-sheet is polyvinylene. Only a small fraction of the PVA is converted to polyvinylene; much of the PVA remains. The conjugation of the polyvinylene is presumed to extend through very long chains, since the material shows strong absorption not only in the visual range but in the near ultraviolet also (L-8, K-22).

For reasons not clearly understood, some *K* polarizers have a num-

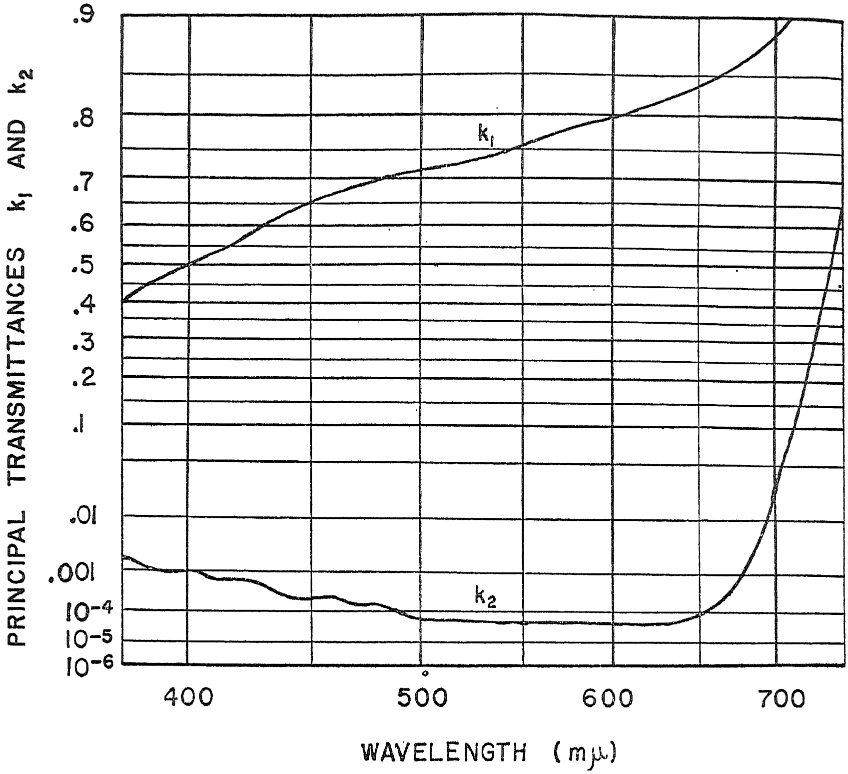


FIG. 4.5. Curves of k_1 and k_2 for KN-36. The vertical scale is linear in log log$(1/k)$.

ber of closely adjacent absorption bands superimposed on the main absorption band. These smaller bands lie mainly in the region from 350 to 500 mμ, as shown by Baxter *et al.* (B-8).

K-sheet is more stable at high temperature and high humidity than is H-sheet. Glass-laminated K-type polarizers can stand a temperature of about 190°F for prolonged periods, and plastic laminated K polarizers can stand about 215°F for fairly long periods. Because of this excellent stability, K-sheet is well suited to use in automobile headlights.

Small pieces of *unsupported* K-sheet — the dichroic sheet proper, with no supporting layer — have been produced experimentally, and have proved to be of use in Lyot filters (Sec. 9.10). Being only about 0.001 in. thick, the material is, of course, very fragile.

4.8. J-Sheet. J-sheet, although superseded 20 years ago by other types of polarizers, has the distinction of being the world's first sheet-type dichroic polarizer and the first polarizer to be put to widespread use by scientists and by the public. When J-sheet became available, scores of new applications of polarized light became practical.

J-sheet is a microcrystalline polarizer. Its invention represented a dramatic break with previous lines of attack. Earlier workers had concentrated their efforts on the attempt to grow dichroic crystals of large size (U-1); but, despite efforts extending over a period of 75 years, little progress had been made, the largest crystals produced being only a few millimeters in width.

In about 1928 Edwin H. Land, then an undergraduate at Harvard College, conceived the idea of employing a statistical array of aligned crystals. He reasoned that if millions of tiny crystals could be brought into approximate alignment their absorption axes also would be nearly parallel, and if the crystals were sufficiently small and were enveloped in a transparent material of similar refractive index, so that light-scattering would be reduced to a low level, the aggregate would behave much like one very large, very thin crystal.

His efforts were successful (L-4, L-8, L-13). He reduced crystals of herapathite to very small particle size, then aligned them, by means of a flow method, for example. A satisfactory enveloping medium, or vehicle, was found, for instance, a solution of cellulose acetate. Shortly before 1930 experimental quantities of the new (J-type) polarizer were produced and the author was privileged to receive and experiment with sample pieces. Within a few years the material was being manufactured on a large scale (see U.S. patents 1,918,848, 1,951,664, 1,955,923, 1,956,867, and various other patents by Land).

Herapathite had been discovered in about 1852 by Dr. William Bird Herapath, an English medical research worker (H-23, H-24). The chemistry of this material (the sulfate periodide of the cinchona alkaloid quinine) has been studied by many investigators; its crystallography was explored in 1937 by West (W-16). The crystals tend to be needle-shaped, with the major principal absorption axis parallel to the long axis of the needle. Thus when a sheet containing millions of such crystals is stretched, the major absorption axis of the sheet is parallel to the stretch direction. The transmission axis is, of course, perpendicular to the stretch direction.

The photometric properties of J-sheet are described in articles by Land and West (L-8) and by Grabau (G-10). The data will not be presented here, since this polarizer has been superseded, principally by H-sheet and K-sheet. Whereas J-sheet, being a microcrystalline polarizer, has some tendency to scatter light (as discussed, for example, by Farwell, F-7), H-sheet and K-sheet, being molecular polarizers, exhibit virtually no scattering. Some Russian work on polarizers containing herapathite crystals has been described by Godina and Faerman (G-7).

4.9. L-*Sheet.* Just before World War II, and also during the early years of that war, a shortage of quinine (and hence of herapathite) threatened. Accordingly Land and his colleagues sought a polarizer that did not require quinine. Several such polarizers were invented. One of these was L-sheet. (Others were H-sheet and K-sheet, discussed in previous sections.)

L-sheet contains myriad aligned, dichroic, organic dye molecules (L-13). It may contain a single black dye, or it may contain several different dyes each of which absorbs in a different portion of the visual range. Land's U.S. patent 2,454,515 mentions the following direct dyes: National Erie Black GX00 (C. I. 581), Amanil Black (C. I. 395), Amanil Fast Black (C. I. 545), Tintex Black, and Logwood; and the following miscellaneous dyes: Niagara Blue 2B (C. I. 406), Solantine Red 8BL (C. I. 278), Niagara Navy Blue BW, Erie Green MT (C. I. 593), Erie Garnet B (C. I. 375), and Solantine Black L (Prototype No. 24).

Other dyes have been listed by Land and West (L-8) and by Zocher (Z-6). Examples are: Congo Red, the orange dye C. I. 374, the blue dye C. I. 518, and the yellow dye C. I. 622.

The dyes may be dispersed in stretched PVA, or in cellophane or other polymeric material.

The density ratio achieved in L-sheet seldom equals the ratios achieved in the H and K polarizers; consequently L-type polarizers have not been used so extensively.

4.10. Other Dichroic, Sheet-Type, Visual-Range, Neutral Polarizers. M-sheet, another polarizer developed by Land and his colleagues, employs aligned microcrystals of types other than herapathite. For example, dibenzacridine periodide may be used (L-8). Such polarizers

were produced commercially during the early years of World War II, but were soon superseded by H-sheet and K-sheet.

Various other sheet-type, dichroic, visual-range, neutral polarizers have been made from time to time by the Polaroid Corporation. Examples are discussed in the patents by Land and his colleagues (for example, U.S. patent 2,454,515) and in articles by Land (L-13) and Land and West (L-8). Some experimental polarizers employed tiny needle-shaped crystals of metal, tellurium, for example; the spectral curves of such polarizers have been published (L-13); the dichroism of tellurium has been studied by Loferski (L-24). Berkman *et al.* (B-14) have summarized efforts to make dichroic polarizers that employ microscopic crystals of gold, silver, and mercury. Cayrel and Schatzman (C-4) have tried using 0.5-μ particles of graphite.

An interesting type of dichroic polarizer is the type discovered by Beilby and later carried forward by Dreyer and his colleagues at Polacoat, Inc. (D-16, D-17, D-18; U.S. patents 2,400,877, 2,432,867, 2,481,830, and 2,544,659). To produce such a polarizer, one imparts a preferred direction to the surface of a sheet of glass or plastic, by rubbing it unidirectionally with filter paper, cotton, or rouge. The affected region extends to a depth of less than 1 μ (Zocher and Coper, Z-7). The sheet is then rinsed and treated with a solution of dichroic molecules having suitable electrical properties, for example, a 0.5-percent solution of methylene blue in ethanol (Z-7). The result is that the adsorbed dye molecules tend to have a preferred direction, and hence exhibit dichroism. In typical polarizers made by Polacoat, Inc., the k_t and k_{vx} values are said to be approximately 0.35 and 0.001 (P-21). In some instances a metal has been used as adsorbed material, with the result that a dichroic *mirror* is achieved as described in Dreyer's patents 2,484,818, 2,776,598. Polaroid Corp. has also produced Beilby layer polarizers; the subject has been reviewed by West (W-19). Articles by Demon (D-5) and by Anderson (A-8) are pertinent also. The Beilby layer method is well suited to making variable-axis-direction polarizers: if the rubbing follows a curved path, successive portions of the polarizer will have progressively different directions of transmission axis. The Polacoat, Inc., "Axis Finder" polarizer employs this principle.

Carl Zeiss Company of Germany has produced a microcrystalline polarizer, called Herotar, containing aligned crystals of herapathite (L-8). That company has experimented also with a polarizer consisting

of one large-area crystal of herapathite; the polarizer was called Mipolar. The Zeiss developments were due mainly to Bernauer (B-15) and have been reviewed by Hasse (H-2, H-4). In about 1926 Zimmern (Z-5) produced polarizers consisting of one crystal, or a few crystals, of iodoquinine sulfate; the material was dissolved and a glass plate was dipped into the solution; as the solvent evaporated, a contiguous group of single crystals each a few millimeters in width was formed. Marks, in U.S. patents 2,104,949 and 2,199,227, describes methods that include an "intensification" step which is said to shorten the time needed to produce large uniform areas of the crystals.

If, using the Dreyer method, one produces a dichroic film on a quartz plate, the resulting polarizer may perform reasonably well even in the ultraviolet range down to 215 mμ, according to McDermott and Novick (M-1). Such polarizers may be used in wide-angle beams. However, in applications where high transmittance and high polarizance are required, the prism-type polarizers discussed in Sec. 5.10 are usually preferred.

4.11. Colored Dichroic Polarizers. To make a *colored*, or *chromatic*, dichroic polarizer, one may employ a stretched sheet that contains oriented molecules of a colored dye. Section 4.9 lists many dyes that have favorable properties. The dyes may be added to the sheet before or after the stretching operation. Satisfactory methods have been indicated by Land and West (L-8) and are described in various patents by Land (see bibliography).

Density ratios of 5 to 15 are achieved in the spectral region in which the dye in question exhibits strong absorption. Illustrative spectral curves are presented in Ref. L-8, in Polaroid Corp. Technical Pamphlet F997, and in the *Polaroid Reporter* for September 1952. See also a 1951 article by Scherer (S-3).

To make a variable-hue polarizer, one may obtain a polarizer containing a single dye and laminate this polarizer, in crossed orientation, with a polarizer containing a dye that absorbs in a different part of the visual spectrum. Such combinations may serve as color-correction filters for use in color photography (G-4, D-12); see Sec. 9.10.

4.12. HR-*Sheet.* HR-sheet, a polarizer for the near-infrared region, contains some of the same ingredients that H-sheet and K-sheet contain: some iodine, polyvinylene, and a large amount of PVA. However,

its spectrophotometric properties (S-15, P-27, B-27) differ from what one might expect from a combination of H and K. Neither H-sheet nor K-sheet absorbs appreciably at wavelengths near 1.5 μ, yet HR has a strong absorption band here, and much dichroism also. One infers that HR contains a special kind of dichroic complex.

Table 4.4 lists the spectral properties of a more or less representative

Table 4.4. Principal transmittances of HR-sheet.

Wavelength (μ)	Principal transmittance	
	k_1	k_2
0.55	0.00	0.00
.60	.01	.00
.65	.05	.00
.7	.10	.00
.8	.30	.00
.9	.50	.01
1.0	.55	.05
1.2	.60	.00
1.4	.65	.00
1.6	.65	.00
1.8	.70	.00
2.0	.70	.00
2.5	.05	.05
3.0	.01	.00

Source: Ref. S-15.

HR polarizer, and Fig. 4.6 shows the variability that may exist among experimental lots produced by slightly different methods and employing different kinds of cover plates. Obviously, the cover plates themselves hurt the performance at wavelengths exceeding 2.3 μ. In general, the HR polarizer performs excellently from 0.7 to 2.3 μ, and in some applications may be useful throughout a larger range, such as 0.6 to 5 μ. HR polarizers can be used for prolonged periods at temperatures as high as 150°F, and briefly at 200°F (Polaroid Pamphlet F976).

HR-sheet was invented at the Polaroid Corporation in the period from about 1943 to about 1951; see U.S. patent 2,494,686 by R. P. Blake, and articles by Land (L-13) and also Blake *et al.* (B-31).

In 1957 the Carl Zeiss Company obtained a German patent on a

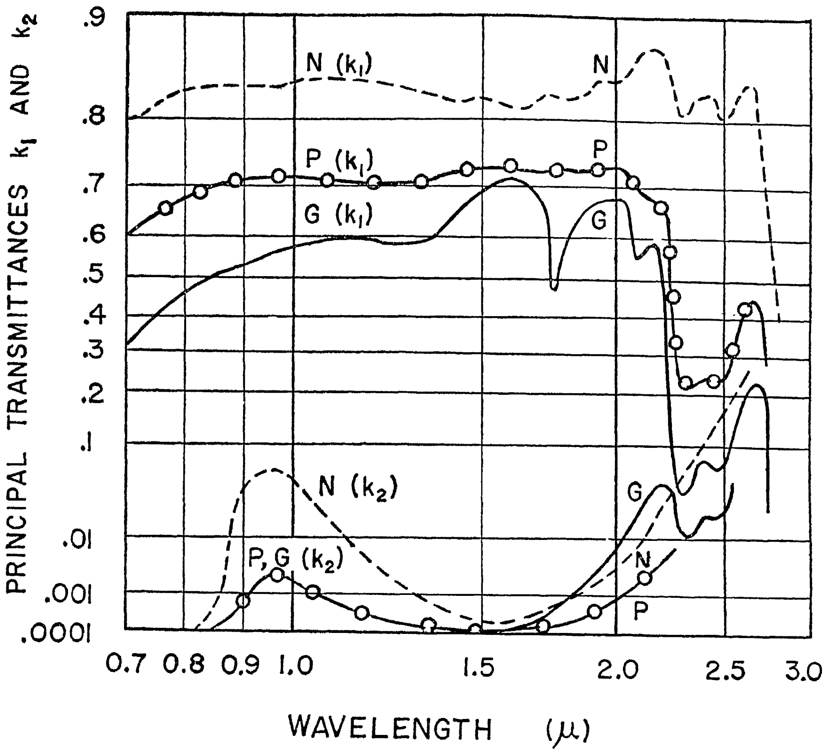


FIG. 4.6. More or less typical curves of k_1 and k_2 for a glass-laminated HR polarizer (G), a plastic-laminated HR polarizer (P), and a naked, unsupported HR film (N). The vertical scale is linear in log $\log(1/k)$.

polarizer having much resemblance to the HR polarizer (German patent 1,015,236).

There are, of course, many other materials that exhibit pronounced dichroism in certain parts of the infrared range, and thus might, perhaps, serve as dichroic polarizers. For example, one might exploit the large dichroism exhibited by an ammonium nitrate crystal at 11 μ and 14 μ; see Newman and Halford (N-2).

5 BIREFRINGENCE POLARIZERS

5.1. Introduction. Birefringence polarizers operate by dividing the incident beam into two completely and orthogonally polarized components, separating them physically, and eliminating one of them. Ideally, the transmitted component is completely polarized and suffers no decrease in intensity. In practice, various effects of reflection, obliquity, and astigmatism hurt the performance. Also, the linear and angular apertures are small, the thickness and weight are great, the cross-sectional shape (a rectangle) is inconvenient, and the cost is high. Consequently such polarizers have been largely superseded by dichroic sheet-type polarizers, though they continue to reign supreme in certain applications, such as those involving ultraviolet light.

The history of the birefringence polarizer goes back to 1690, when Christian Huyghens, experimenting with calcite crystals, discovered polarized light (H-42). The Nicol prism was invented in 1828, and superior designs were perfected subsequently. Some of the greatest minds in crystallography and microscopy have been applied to this subject. The early history of the birefringence polarizers is recorded in anonymous works published in 1819 (A-10) and 1843 (A-11), and in works by Gange (G-2), Johannsen (J-18), Thompson (T-5), and Tutton (T-12). More recent developments have been reviewed by Archard (A-19, A-20) and Twyman (T-13).

Because refractoanisotropy plays the central role not only in birefringence polarizers but also in retarders, a brief review of the main parameters is presented below. Detailed accounts appear in works by

Born (B-41), Bouasse (B-44), Jenkins and White (J-9), Ditchburn (D-10), Johannsen (J-18), and Wahlstrom (W-1).

5.2. Refractoanisotropy and Birefringence. When a monochromatic ray passes from an evacuated region into a cube of glass, the speed of propagation (ray speed) within the glass is less than the speed in vacuum. The ratio of the latter speed to the former is called n, the refractive index of the glass. The value of n remains the same irrespective of which face of the cube is taken as entrance face, irrespective of the obliquity of the ray, and irrespective of its direction of vibration. Consequently the glass is called refractoisotropic.

The general body is refracto*anisotropic.* When a ray strikes a cube cut from such a body, *two* refracted rays are produced, each having its own direction (ray direction), own set of wave fronts, and own wave normal (normal to the wave front). For each of the two refracted rays, two kinds of speed may be defined: *ray speed* and *normal speed.* The ray speed is simply the speed of propagation along the ray direction (Fig. 5.1). The normal speed is the product of (1) the ray speed and

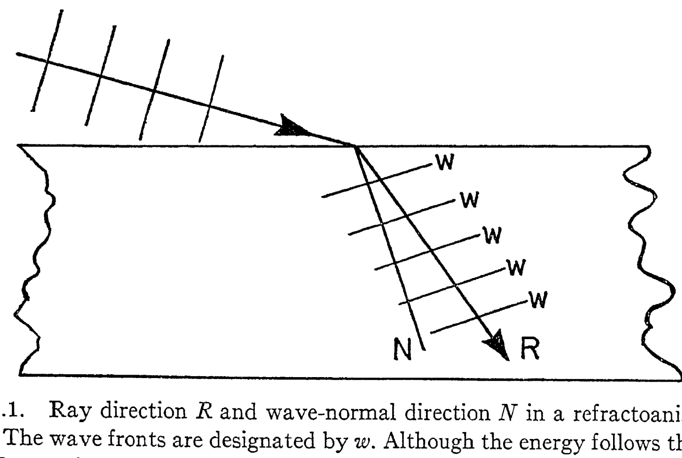


FIG. 5.1. Ray direction R and wave-normal direction N in a refractoanisotropic body. The wave fronts are designated by w. Although the energy follows the direction R, one finds that the direction N plays the main role in the graphic and algebraic descriptions of refractive index.

(2) the cosine of the angle between the ray direction and the wave normal. The refractive index of the body with respect to a given ray is defined as the ratio of the speed in vacuum to the *normal speed* of the given ray in the body. (Many textbooks fail to present a clear definition of the refractive index; they leave the reader free to conclude

that the ratio in question is the ratio of speed in vacuum to *ray speed.*)

The direction one usually deals with in problems involving birefringence is the *wave-normal direction*, and the speed one usually deals with is the *normal speed.* Only by employing these "normal" parameters can one describe the main refraction properties of the body in a simple manner.

The two refracted rays produced when the unpolarized beam strikes the cube obliquely are found to be polarized, and in fact orthogonally polarized. In some instances the two polarization forms are linear, and the body is said to be linearly birefringent. Other bodies produce circular or elliptical forms, and are called circularly or elliptically birefringent. To simplify the discussion, we shall confine our attention to *linearly* birefringent bodies; also we shall assume that the bodies are homogeneous and exhibit no scattering and no absorption.

When the ray is incident on a different face of the linearly birefringent cube, or when the ray is incident at different obliquity, each of the refracted rays is changed: its ray direction and ray speed change, and its wave normal and normal speed change. *The key direction here is the vibration direction:* if it remains the same, the normal speed remains the same; if it changes, the normal speed changes. By experimenting with a polarized refracted ray having, successively, all possible vibration directions, one arrives at a maximum value of refractive index, called the major principal refractive index γ. Similarly one finds a minimum value, the minor principal refractive index α. The family of wave-normal directions associated with γ defines a plane whose normal is Z, the major principal axis of refraction. Similarly, the wave-normal directions associated with α define a plane whose normal is X, the minor principal axis of refraction. The X- and Z-axes are perpendicular to one another. The direction perpendicular to both is called Y, the intermediate principal axis of refraction; associated with it is the intermediate principal refractive index β. (The products of c, the speed of light in vacuum, and the reciprocals of the principal indices α, β, γ are called the *principal normal speeds* v_α, v_β, and v_γ; for example, $v_\alpha = c/\alpha$.)

We are now in a position to define the *refraction indicatrix*, the most succinct summary of the body's refraction properties. It is the ellipsoid whose semiaxes are parallel to X, Y, and Z and have lengths equal to α, β, and γ. It serves as a three-dimensional model from which the two indices associated with any given wave-normal direction within

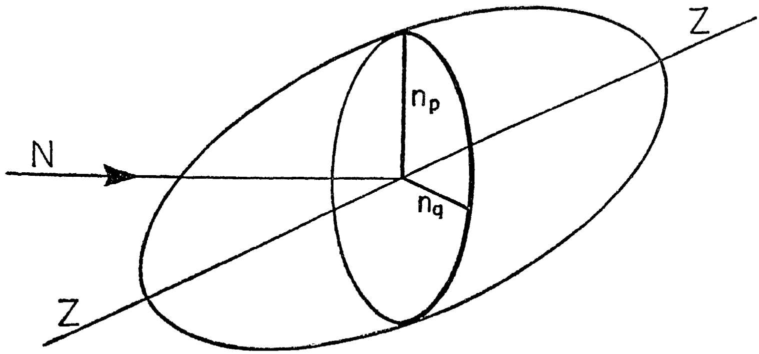


FIG. 5.2. Method of finding the two (nonprincipal) refractive indices n_p and n_q, for the given wave-normal direction N and the given refraction indicatrix.

the body may be found. The procedure, suggested by Fig. 5.2, is to cut the indicatrix with a central plane that is perpendicular to the given wave-normal direction; the plane intersects the indicatrix in an ellipse, and the lengths of the ellipse's major and minor semiaxes are proportional to the two pertinent (nonprincipal) refractive indices n_p and n_q. (The situation is much like that concerned with the absorption indicatrix of Chapter 4. One of the main differences is that the refraction indicatrix usually departs but slightly from a sphere, whereas the absorption indicatrix may be very far from spherical.)

The indicatrix may be positive or negative (that is, prolate or oblate). It is called positive if

$$\frac{1}{\alpha^2} - \frac{1}{\beta^2} < \frac{1}{\beta^2} - \frac{1}{\gamma^2}.$$

An equivalent criterion is that the optic axial angle $2V$, defined in a later paragraph, be less than 90°.

The birefringence of the general refractoanisotropic body is not one quantity, but three, namely, the three differences between the major principal refractive indices: $\gamma - \beta$, or $J_{\gamma\beta}$; $\beta - \alpha$, or $J_{\beta\alpha}$; and $\gamma - \alpha$, or $J_{\gamma\alpha}$. They may be called respectively the first, second, and over-all coefficients of birefringence.

In any homogeneous body exhibiting essentially no absorption in the visual range, one may find an *isoaxis of refraction*, that is, a wave-normal direction for which the two pertinent refractive indices n_p and n_q are identical. In the general absorboanisotropic body there are two such axes, called *optic axes;* they make equal angles V with the

Z-axis, and the double angle $2V$ is called the *optic axial angle.* Because the body has two isoaxes of refraction, it is called *refractobiaxial.*

Some bodies have only one isoaxis of refraction; that is, in some bodies the two isoaxes are identical. Such a body is called refractouniaxial. It has only two principal refractive indices, called ω ("ordinary") and ϵ ("extraordinary"). If the indicatrix — now an ellipsoid of revolution — is positive, $\epsilon > \omega$; if it is negative, $\epsilon < \omega$. The difference $|\epsilon - \omega|$ is the *birefringence* of the uniaxial body, and may be called J. (Many authors use birefringence to mean $\epsilon - \omega$; this is a signed quantity, and is positive or negative depending on whether the indicatrix is positive or negative. Many authors represent the birefringence by the symbol Δn.) When the wave normal of a given ray within the body is oblique to the optic axis, one of the two indices continues to be equal to the principal index ω, and the other lies between ω and ϵ; it may be called ϵ'. A polarized ray that is governed by the uniaxial body's ordinary index is called an ordinary ray (O-ray), and a ray governed by ϵ (or ϵ') is called an extraordinary ray (E-ray).

When a beam is incident normally on a birefringent sheet, the two polarized components within the sheet have, of course, the same wave-normal direction (the direction perpendicular to the sheet). Consequently we are then concerned with but a single cross section of the refraction indicatrix, namely, the cross section that is parallel to the sheet.

The term planorefraction may be given to the refraction process concerned. In the general case the cross section is elliptical. The principal axes of the ellipse may be called the major and minor axes (or the slow axis S and the fast axis F); the lengths of the major and minor semiaxes may be called the major and minor indices γ_p and α_p, and the magnitude of the difference between these may be called the *planobirefringence* J_p.

5.3. Materials Used. Birefringence polarizers usually employ calcite ($CaO \cdot CO_2$), a transparent, refractouniaxial crystal belonging to the hexagonal crystallographic system and having a negative (oblate) indicatrix. The over-all coefficient of birefringence is large: for 5983-A sodium light

$$J \equiv |\epsilon - \omega| = |1.486 - 1.658| = 0.172$$

(A-7, S-19); for 3400 and 7600 A, the J values are 0.195 and 0.167. These values are so large that even beams having great angular width

may be polarized successfully. The material is transparent from about 2400 A in the ultraviolet to about 1.8 μ in the infrared (B-46); outside this range it exhibits strong absorption and, in fact, some dichroism.

Sodium nitrate ($NaNO_3$) is another pertinent material. Its ϵ and ω values are 1.3369 and 1.5854; thus $J = 0.2485$, which is larger than for calcite. Quartz, a positive refractouniaxial crystal, has a birefringence

$$J \equiv |\epsilon - \omega| = |1.553 - 1.544| = 0.009;$$

this is very small, and accordingly quartz is seldom used in polarizers, though often used in retarders. Various other birefringent crystals are listed in the *American Institute of Physics Handbook* (A-7), Sec. 6-b; see also S-19.

Many birefringence polarizers employ a thin layer of isotropic material between two birefringent prisms. The isotropic material must satisfy many requirements (discussed at length by Bouriau and Lenoble, B-46): it must have a suitable refractive index; it must be transparent and chemically stable; usually it must serve as a cement also. Canada balsam is the usual choice; its index is 1.55, which is approximately midway between those of calcite. In polarizers intended for use below 330 mμ a different cement must be used, since Canada balsam is opaque here. One procedure is to use gedamine, a butyl alcohol solution of urea formaldehyde; its index is approximately 1.52 and it is transparent to wavelengths as short as 250 mμ (B-46). Another procedure is to use a thin air gap.

The following sections describe various specific designs, starting with those of principal current interest.

5.4. Ahrens Polarizer. The Ahrens polarizer, invented in 1886, consists of three prisms of calcite cemented together with Canada balsam to form a rectangular block (Fig. 5.3). The length/width ratio of the block is approximately 1.9. When unpolarized light is incident on the face containing the apex of the central piece of calcite, the E-ray passes straight through. The O-ray, however, suffers total internal reflection at the oblique (cement) interface (since the ordinary index of calcite far exceeds the index of the cement) and is absorbed by the black coating on the sides of the polarizer.

The device has excellent characteristics. It has very high polarizance, perhaps exceeding 0.999,99. Relative to various other birefringence polarizers it has a large linear aperture (relative to the length), and a

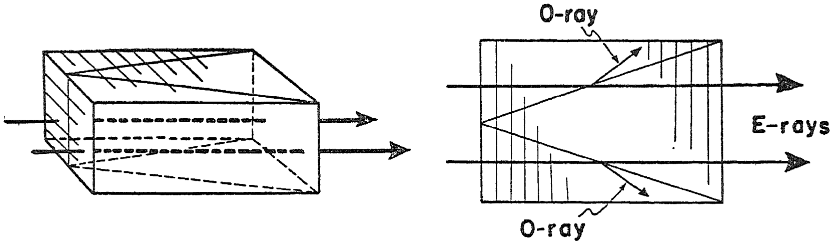


FIG. 5.3. Ahrens polarizer. The direction of the optic axis is indicated by the hatch marks. The four lateral faces are covered with black paint.

large acceptance angle. The faces used are squarely perpendicular to the incident ray. The device is used in polarizing microscopes.

The dividing line seen across the center of one face is sometimes undesirably conspicuous, but may be made less so by cementing a thin glass cover slip to this face.

The design has been described in detail by Johannsen (J-18). (In some instances, a different choice of optic-axis direction is used.) Ahrens polarizers are manufactured by the Crystal Optics Company of Chicago, Illinois.

5.5. Wollaston Polarizer; Rochon Polarizer. The Wollaston polarizer produces *two* orthogonally polarized beams. It divides the incident light into two polarized components, deviates them oppositely (Fig. 5.4), and transmits both. The two pieces of calcite are joined by a layer of Canada balsam. The total deviation angle ϕ depends on the choice of wedge angle θ (and varies slightly with wavelength). The device is compact and square ended, and it performs excellently in most applications where two orthogonally polarized beams are required.

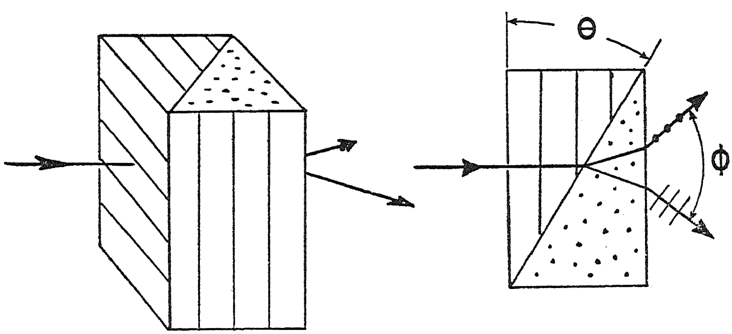


FIG. 5.4. Wollaston polarizer.

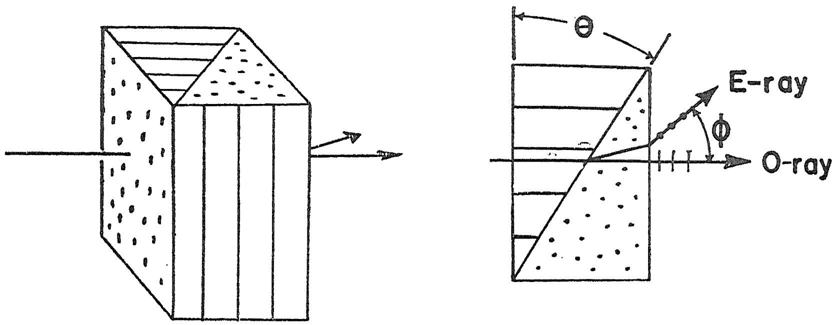


FIG. 5.5. Rochon polarizer.

The Rochon polarizer (Fig. 5.5), invented in 1783 (W-22), bears much resemblance to the Wollaston. Indeed, if the diagram of the Wollaston is turned through 90° it resembles the diagram of the Rochon, except for details as to shape. Again both of the components are transmitted. The *O*-ray proceeds straight through, and the *E*-ray is deviated through a moderately large angle ϕ that depends on the choice of θ.

5.6. Glan-Foucault Polarizer; Taylor Modification. The Glan-Foucault polarizer was designed to polarize ultraviolet light (and succeeds in polarizing visible and infrared light also). The two calcite prisms are separated by an air gap (Fig. 5.6). In each prism the optic axis is

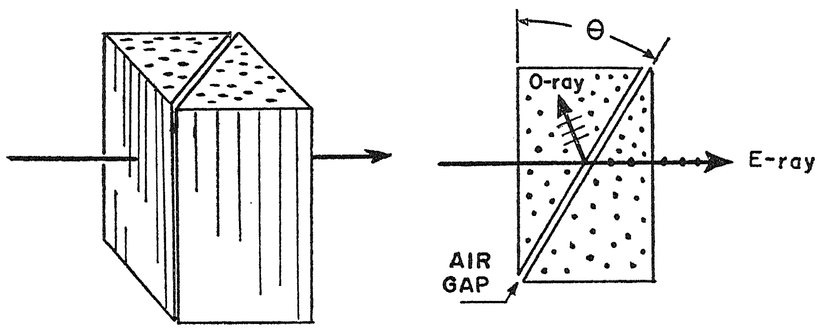


FIG. 5.6. Glan-Foucault polarizer.

perpendicular to the beam and to the upper face. The angle θ is about $38\frac{1}{2}°$, according to Archard and Taylor (A-19), and the acceptance angle is about 7°. The transmitted component is the *E*-ray. The reflec-

tion losses suffered by the wanted component are large, because of the two additional air-calcite interfaces; hence the k_1 value is only about 0.50 (A-19).

To provide a higher value of k_1, Archard and Taylor in 1948 (A-19) proposed a modified design in which the optic axis in each prism is parallel to the entrance face and also to the upper face (Fig. 5.7).

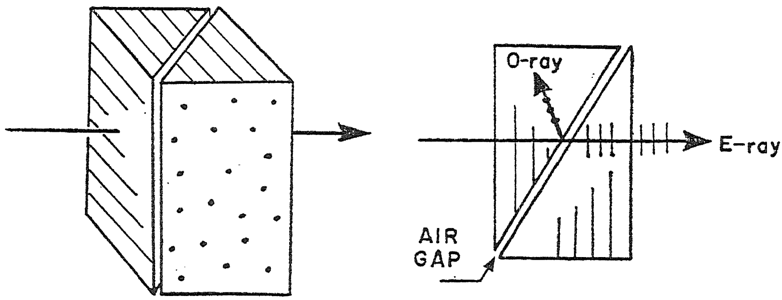


FIG. 5.7. Taylor modification of Glan-Foucault polarizer.

Reflection losses at the oblique surfaces are reduced so much that $k_1 \cong 0.90$.

5.7. Nicol Polarizer. The Nicol polarizer, invented in 1828 by the Scottish physicist William Nicol (N-5), was the first of the highly successful birefringence polarizers. For over a century the term *Nicol* was virtually synonymous with *polarizer*.

No detailed description of the Nicol will be presented here. The polarizer was largely superseded several decades ago; in visual-range work the Ahrens, Glazebrook, and Rochon polarizers — or the new types of dichroic, sheet-type polarizer — are usually preferred; in ultraviolet investigations the Glan-Thompson and modified Glan-Thompson polarizers are preferred.

The design of the Nicol is difficult to describe and difficult to grasp. Johannsen (J-18) devotes seven pages to the matter. The general features are suggested by the sketches of Fig. 5.8. Various minor changes in design have been made from time to time.

The performance of the Nicol leaves much to be desired. Because of the obliquity of the entrance and exit faces, (*a*) the wanted component does not proceed straight through, but is displaced laterally, (*b*) the emerging beam is actually elliptically polarized, not linearly polarized (though, to be sure, the ellipticity is extremely small), and (*c*) astig-

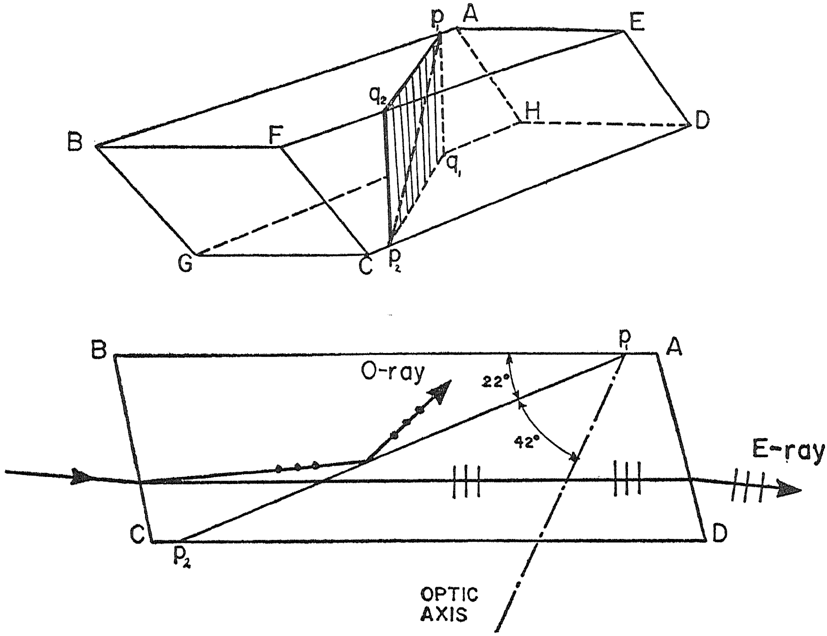


FIG. 5.8. Nicol polarizer: (*a*) a rhombohedron of calcite prepared largely by cleavage, in which $p_1q_1p_2q_2$ is the plane in which a saw cut is made and Canada balsam is inserted; (*b*) the plane *ABCD* and the optic axis, which lies in this plane. The four lateral faces are coated with black paint.

matism (with respect to light approaching the polarizer from a nearby source) is appreciable. The acceptance angle is relatively small, being about 25 to 28° (J-9, J-18).

Various investigators have proposed minor changes in the original design. A modified form proposed in 1908 by Halle (D-7) is currently produced by Bausch and Lomb, Inc., Rochester, New York.

5.8. Other Polarizers Containing Calcite. Of the many other kinds of polarizers that contain calcite, only a few will be mentioned here. The others are described in the works listed in Sec. 5.1.

The Glazebrook polarizer resembles half of an Ahrens polarizer, described in Sec. 5.4; if that polarizer is divided in half by a plane perpendicular to the optic axis (Fig. 5.3), each half is equivalent to a Glazebrook polarizer. Obviously the length/width ratio (approximately 3.8) is much greater than that of the Ahrens polarizer.

The Foucault prism is somewhat similar to the Nicol prism in that

the entrance and exit faces are oblique (J-18, D-10). It is similar to the Glan-Foucault polarizer in that an air gap, instead of Canada balsam, is used. The acceptance angle is small (about 8°), and the total transmittance is low.

The Prazmowski polarizer, sometimes called the Hartnack-Prazmowski polarizer, may be thought of as a Nicol polarizer modified in such a way that the entrance and exit faces are perpendicular to the beam.

The Glan-Thompson polarizer is much like the Glan-Foucault polarizer, but has a much greater length/width ratio and a much greater acceptance angle, namely, about 40° (J-9).

Other designs have been proposed by Madan (J-18, M-8), Bertrand (B-16), Senarmont (D-7), and Bouhet and Lafont (B-45).

In 1931 Cotton (C-30) proposed a polarizer consisting of a single 45° prism of calcite (Fig. 5.9). Both the *O*-ray and the *E*-ray emerge, and both are deviated through large angles.

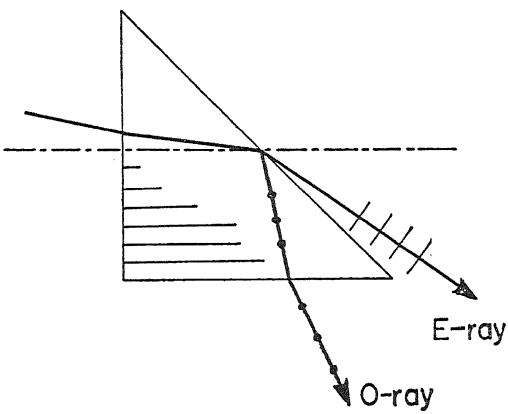


FIG. 5.9. Cotton polarizer.

In 1957 Bouriau (B-46) produced a polarizer having much resemblance to the Glazebrook polarizer, but employing gedamine (Sec. 5.3) instead of Canada balsam.

Some designs of polarizers have employed one piece of calcite and one or more pieces of other material. In 1872 Talbot proposed a design employing one calcite prism and one glass prism; somewhat similar designs were proposed by Leiss in 1897 and by von Lommel in 1898 (J-18). In 1901 von Federov designed a polarizer consisting of a calcite

hemisphere (with optic axis parallel to its plane face) embedded in a glass block having an intermediate index (J-18). In 1936 Osipov-King (O-4) designed an ultraviolet polarizer employing a calcite prism and a thin plate of silica.

The Fuessner polarizer, invented in 1844 (F-20), employs a thin plate of calcite mounted obliquely between two prisms (wedges) of glass. The same is true of the Sang polarizer and a certain variety of Bertrand polarizer (J-18). West in 1948 (U.S. patent 2,447,828) invented a polarizer consisting of an equilateral glass prism whose base is in contact with a thin plate of calcite; light enters and emerges through the sides; the unwanted component emerges from the calcite base plate.

A few polarizers consisting mainly of sodium nitrate were made by Bouhet and Lafont in 1949 (B-45). Bertrand, also, has used sodium nitrate (J-18). The Polaroid Corporation has experimented with polarizers consisting of a thin sheet of birefringent organic-polymeric material (for instance, stretched Cronar) mounted obliquely in a glass box containing a liquid whose index matches the major planorefractive index of the sheet; thus one (polarized) component of the light is reflected and the other is transmitted.

5.9. Scattering-Birefringence Polarizers. Some birefringence polarizers eliminate the unwanted component not by one clean deviation but by scattering this component in many directions. The elimination is not complete, since a small fraction of the scattered light is likely to proceed straight ahead, along with the wanted component. Nevertheless, reasonably successful polarizers of this *scattering-birefringence* type have been made.

Some of the experimental devices made by the Polaroid Corporation employed myriad tiny crystals of highly birefringent material such as urea guanidine or caffein. The crystals were embedded in an isotropic plastic sheet whose index was approximately the same as *one* of the principal indices of the crystal. The polarizers were reasonably successful; they had moderately large density ratios and had, of course, a cloudy appearance. Further details are given in patents 2,122,178, 2,123,901, and 2,123,902 by Land.

In 1955 Yamaguti (Y-1) prepared a polarizer consisting of a layer of sodium nitrate situated (in fact, *grown in situ*) between two plates of high-index glass. In the design in question, the sodium nitrate con-

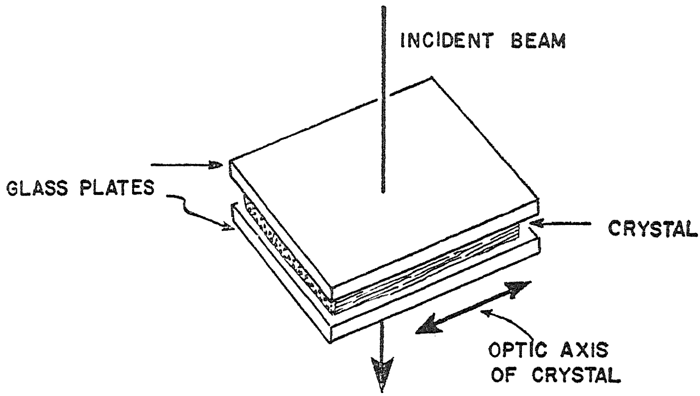


FIG. 5.10. Yamaguti's scattering-birefringence polarizer.

sists essentially of a single crystal whose optic axis is parallel to one long edge of the assembly (Fig. 5.10). The glass plates, whose index matches the ordinary index of the crystal, are polished on one face and roughened on the other, the latter being in contact with the crystal. When unpolarized light is incident perpendicularly on the three-layer sandwich, the *O*-ray passes straight through, since it encounters no discontinuity in refractive index. The *E*-ray, however, encounters a large discontinuity and hence is scattered in all directions at the rough interfaces.

5.10. Applicability to Ultraviolet and Infrared Radiation. Nearly all of the polarizers discussed above are applicable throughout the visual range of the spectrum. Those that employ gedamine cement or use an air gap are useful down to about 240 mμ — or even shorter wavelengths if the length of the polarizer is kept very small, so that absorption by the calcite remains small (B-46).

As regards performance in the near infrared, Ellis and Bath (E-13) have found that a Glan-Thompson polarizer 1 in. long performs well from 0.7 to 3.0 μ and from 4.4 to 4.9 μ. Kaye (K-5) reports that a 1-in. Nicol polarizer is useful from 0.7 to 2.35, from 2.47 to 2.52, from 2.65 to 2.70, and from 2.80 to 2.95 μ. Duverney and Vergnoux (D-25) have compared many types of infrared polarizers.

It is apparent that birefringence polarizers are still first choice for use in the ultraviolet range, despite their small size, great bulk, and high cost.

6 REFLECTION POLARIZERS

6.1. Introduction. Here we discuss reflection polarizers, and also certain other types that are of occasional interest.

A reflection polarizer may consist of a single reflecting surface (for example, the surface of a pond, or of a plate of black glass), a pair of reflecting surfaces (such as the two surfaces of a transparent glass plate), or a large number of surfaces, as in the pile-of-plates polarizer. The last-named device is of interest because both the polarizance and the major principal transmittance may be very high; such polarizers are used with great success in infrared spectrophotometry.

The name *reflection polarizer* is used irrespective of whether the reflected beam or the transmitted beam is the one that is retained and used. In either case, the discrimination between two orthogonal polarization forms is accomplished by a process involving reflection at an oblique surface.

6.2. Polarizance of a Single Surface. When a monochromatic beam is incident obliquely on a smooth flat surface of glass or other dielectric material (Fig. 6.1), the reflected beam will be found to be partially linearly polarized, with the dominant vibration direction perpendicular to the plane of incidence (the plane containing the beam and the normal to the surface). The refracted beam also is partially linearly polarized, although to a lesser extent, ordinarily; its dominant vibration is parallel to the plane of incidence.

The equations governing the reflection coefficients of a single surface of a dielectric body situated in vacuum were originally worked out by

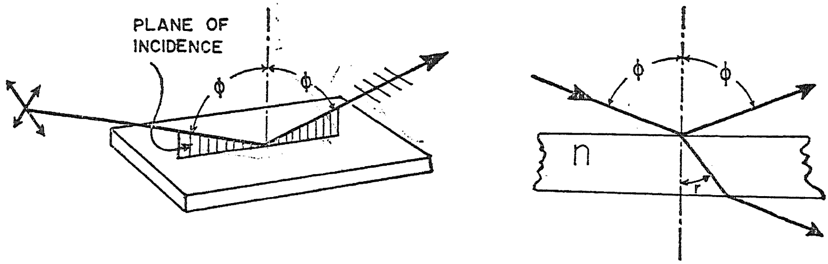


FIG. 6.1. Symbols used in discussion of polarization by reflection from a glass plate G.

Fresnel, and have been discussed by many authors, including Ditchburn (D-10), Tuckerman (T-11), and Jenkins and White (J-9). (If the dielectric body is in contact with a metallic layer, the situation is much more complicated; see Harrick (H-14), and other references listed by him.) Let ϕ be the angle of incidence, r the angle of refraction, and n the refractive index. Then if ρ_{90} is the reflection coefficient with respect to linearly polarized incident light whose electric vibration is *perpendicular* to the plane of incidence, and if ρ_0 is the coefficient for vibrations that are *parallel* to this plane, Fresnel's equations yield:

$$\begin{aligned} \rho_{90} &= \frac{\sin^2(\phi - r)}{\sin^2(\phi + r)}, \\ \rho_0 &= \frac{\tan^2(\phi - r)}{\tan^2(\phi + r)}. \end{aligned} \tag{6.1}$$

Figure 6.2 presents graphs of these quantities (assuming $n = 1.50$). The curve for ρ_{90} rises monotonically, from 0.04 to 1.00. The curve for ρ_0 is more interesting: it drops to zero when ϕ is such that $\tan^2(\phi + r) = \infty$, that is, when $\phi = \arctan n$, which occurs when the angle between the reflected beam and the refracted beam is 90°. This special angle is called *Brewster's angle*, or the *polarizing angle*. When, for example, n has the values 1.4, 1.5, 1.6, or 2.0, Brewster's angle is 54.5°, 56.3°, 58.1°, or 63.4° respectively.

If the incident beam is unpolarized, the degree of polarization V of the reflected beam (defined in Chapter 1) is

$$V = \frac{\rho_{90} - \rho_0}{\rho_{90} + \rho_0}.$$

This quantity is zero when $\phi = 0°$ or 90°. In principle, it has the value 1.0 when ϕ is Brewster's angle, but in practice the value falls slightly

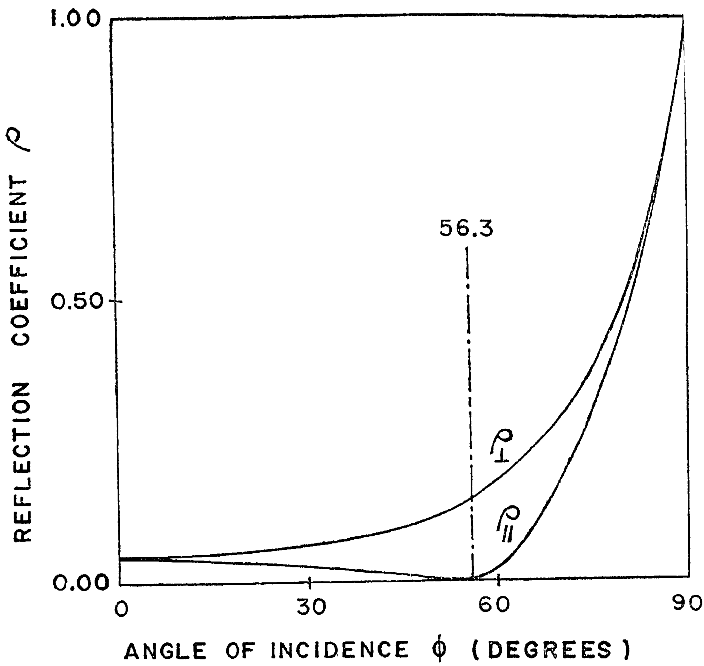


FIG. 6.2. Reflection coefficient of a single surface of a dielectric body in vacuum; the refractive index n is assumed to be 1.50.

short of 1.0 for reasons relating to surface films, surface strains, and so forth. Also, the polarization form may correspond to a very slender ellipse, rather than a straight line; see Ditchburn (D-10), Laurence (L-15), and Mussett (M-30). When $n < 2.0$, the intensity of the reflected beam is low; but if $n \cong 4.0$, as for germanium with respect to infrared radiation of wavelength 2–12 μ, the intensity is approximately 40 percent of that of the incident, unpolarized beam (Edwards and Bruemmer, E-7).

The refracted beam, too, is partially linearly polarized. Since, by hypothesis, the body in question does not absorb, the two transmission coefficients are easily derived from the two reflection coefficients, and are $(1 - \rho_0)$ and $(1 - \rho_{90})$.

6.3. Polarizance of a Pile-of-Plates Polarizer. To achieve high polarizance and also a high transmittance in a reflection polarizer, one usually employs a succession of at least six dielectric plates, arranged in a pile, and then makes use of the *transmitted* beam. If the m plates

are parallel-sided, nonabsorbing, nonbirefringent, and nonscattering, if they are sufficiently thick and well separated that no harmful interference effects occur, and if they are mounted at Brewster's angle, the polarizance P of the combination is easily computed from equations due to Stokes (S-30), Wood (W-26), Provostaye and Desains (P-33), and Tuckerman (T-11), and converted to consistent forms by Bird and Shurcliff (B-28):

$$P = \frac{1 - (2n/n^2 + 1)^{4m}}{1 + (2n/n^2 + 1)^{4m}} \tag{6.2}$$

and

$$P = \frac{m}{m + (2n/n^2 - 1)^2}. \tag{6.3}$$

Equation (6.2) is applicable when only the directly transmitted component reaches the detector, as for example when the reflected rays are blocked off by suitable diaphragms or slit jaws.

Equation (6.3) applies when, in addition to the directly transmitted component, all the rays that have been reflected an even number of times reach the detector. Such rays tend to be relatively rich in the unwanted component, and accordingly this equation leads to polarizance values appreciably lower than those given by Eq. (6.2). Consider, for example, a six-plate (twelve-surface) silver chloride polarizer. Here $m = 6$ and (for 2-μ light) $n = 2.006$. Inserting these values in Eqs. (6.2) and (6.3) one obtains polarizances of 99.0 and 77.3 percent respectively (B-28).

In practice, neither equation represents an absolute limit. The polarizance may be even greater than implied by Eq. (6.2) if all normal precautions are taken and if, in addition, the angle of incidence slightly exceeds the Brewster angle (S-30, W-26, B-28). On the other hand, the polarizance may be even less than implied by Eq. (6.3) if dust, birefringence effects, and so forth are present. Charney (C-13) has pointed out the complications that arise, in measuring dichroic ratios of oriented polymers, if the polarizer's polarizance is appreciably less than unity.

A "zero-displacement" design that assists the rejection of reflected rays has been explored by Makas and Shurcliff (M-9). Instead of arranging the six plates in a single stack, as in Fig. 6.3*a*, they arranged them in two oppositely sloping groups of three plates each, as in Fig. 6.3*b*. Besides eliminating a majority of the reflected rays, this

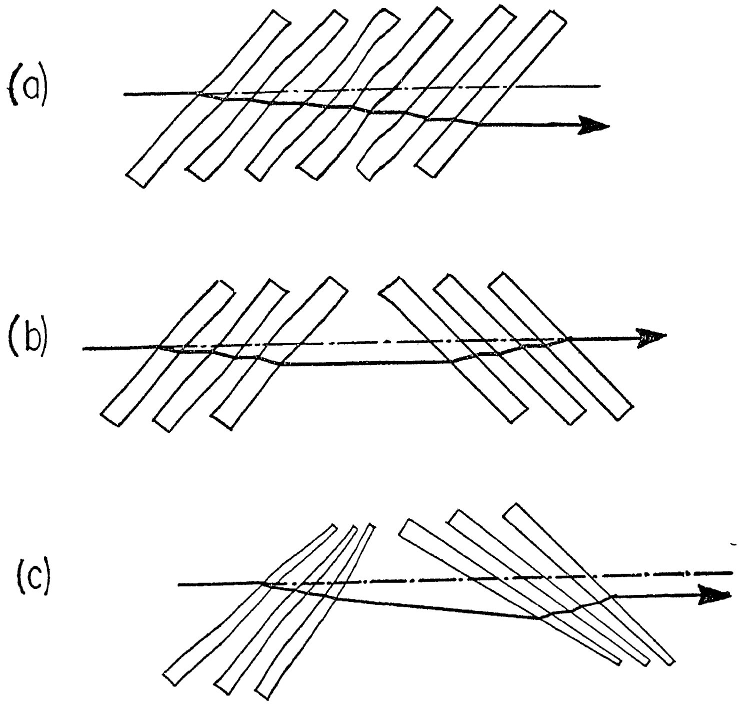


FIG. 6.3. Designs of a six-plate polarizer: (*a*) plates arranged in a single group; (*b*) plates arranged in two groups; (*c*) two groups of plates that are wedged and fanned.

scheme avoids displacement of the emerging beam with respect to the incident beam and thus makes it practical to use plates that are comfortably thick, so that commercially available plates may be used *without change*. (Efforts to reduce the thickness tend to produce birefringence and cloudiness, and make the plates less flat and less rugged.) A more refined scheme, proposed by Bird and Shurcliff (B-28), is indicated in Fig. 6.3*c*. Here the individual plate is very slightly wedge-shaped (angle about 1°), and each group of plates is "fanned" slightly. As a consequence, virtually none of the reflected rays follows the same path as the main transmitted beam.

6.4. Examples of Reflection Polarizers. Several investigators have experimented with pile-of-plates polarizers for use in the near infrared region (N-2, W-29, M-9, B-28). Usually, silver chloride plates are used (Kremers, K-18) and the polarizer may be used throughout the

infrared range from about 1 μ to 20 μ. Typical designs provide a polarizance of perhaps 85 to 95 percent, but when special precautions are taken (M-9, B-28) a polarizance of 99 or even 99.5 percent may be achieved. To measure such a high polarizance is difficult, as explained by Bird and Shurcliff (B-28). In a sense, the polarizer consists of the *combination* of oblique plates and the various slits, diaphragms, and so forth, that are present in the optical system and block some fraction of the unwanted rays that would tend to degrade the polarizance. To cross this bulky combination with another similar combination would be very difficult.

Silver chloride polarizers usually exhibit some scattering, and hence have low transmittance and low polarizance for short-wavelength light, that is, visible light. Also, the silver chloride plates are subject to photolysis and hence must be protected from intense actinic light.

In some instances, very thin (1–5-μ) films of selenium have been used in infrared polarizers (P-15, E-8, C-25, C-26). Such thinness insures that deviation effects and interference effects will be negligible. Polarizance as high as 99.7 percent has been claimed for an experimentally produced polarizer due to Conn and Eaton (C-25, C-26, B-56); 96 percent polarizance is claimed for a unit produced commercially by Hilger and Watts, Ltd.

Various other materials, such as calcium fluoride and KRS-5, may be used in infrared polarizers (Lagemann and Miller, L-1). Elliott *et al.* (E-8, E-9) have used alternate layers of thallium iodide and sodium fluoride, whose indices are 2.5 and 1.3 respectively. Harrick (H-15) achieved a very high degree of polarization (approximately 99 percent) by employing two successive reflections from a germanium-mercury interface. Mitsuishi *et al.* (M-23) have found that a polarizer consisting of 15 layers of thin polyethylene film performs well in the range from 3 to 200 μ.

Visual-range polarizers may employ glass or various organic polymers. A polarizance of 95 percent is easily achieved with a fanned stack of 15 microscope slides. Ultraviolet polarizers may consist of vycor, fused silica, or quartz.

Useful polarizers having only two reflecting surfaces have been made. In 1831 Norrenberg built a polarizer consisting of two parallel, widely separated plates of black glass; the beam encountered the first plate at Brewster's angle, and the reflected component then encountered a second plate mounted parallel to the first (P-10). In 1948

Pfund employed an infrared polarizer consisting of two plates of selenium (P-15); recently, Edwards and Bruemmer have demonstrated the superiority of two plates of germanium (E-7). Meier and Günthard (M-17) also have demonstrated the effectiveness of polarizers employing germanium.

Even x-rays can be polarized to a high degree by a reflection method. In 1956 Chandrasekaran (C-11), using Bragg reflection at 45°, achieved a degree of polarization of 99.8 percent. Excellent results were obtained also by George (G-5).

6.5. Scattering Polarizers. Dust particles, the molecules of a gas, and even the atoms of a perfect crystal can produce polarization by means of scattering. The scattering produced by a volume of gas is, of course, slight; the intensity is proportional to $1/\lambda^4$, as explained by Rayleigh. The degree of polarization may be large; indeed, if the scattering material is a gas, if the light examined is the light that is scattered through 90°, and if the experiment is designed so that little *multiple* scattering occurs, the degree of polarization may be as great as 99.5 percent (D-10). For large molecules such as N_2 and CO_2, the polarization may amount to 90 to 96 percent. For highly polar molecules, and for dust or other relatively large particles, it may be very small.

The dominant vibration direction in the scattered beam is, of course, the direction that is perpendicular to the incident beam and to the scattered beam in question (Fig. 6.4). Thus if the initial unpolarized beam B is propagating horizontally, a ray S_h that is scattered at 90° in a horizontal plane has a vibration direction that is vertical, and a ray S_v that is scattered vertically has a horizontal vibration direction.

The theory of scattering, as by the atmosphere, has been studied at length by Chandrasekhar (C-8), Chandrasekhar and Elbert (C-10), van de Hulst (V-1), Lenoble (L-18), and Kuscer and Ribaric (K-23). The experimental work on the polarization of skylight by scattering has been summarized by Sekera (S-8) and Chandrasekhar and Elbert (C-10). Recent measurements made at many different wavelengths (including ultraviolet) and at different times of day, from sunrise to sunset, have been reported by Coulson, Sekera, and others (C-31).

Scattering methods may be used to polarize x-rays and gamma-rays. In experiments made by McMaster and Hereford (M-4), a radiocobalt source emitted 1.1- and 1.3-Mev photons which were then subjected to Compton scattering by a small block of copper. The radiation

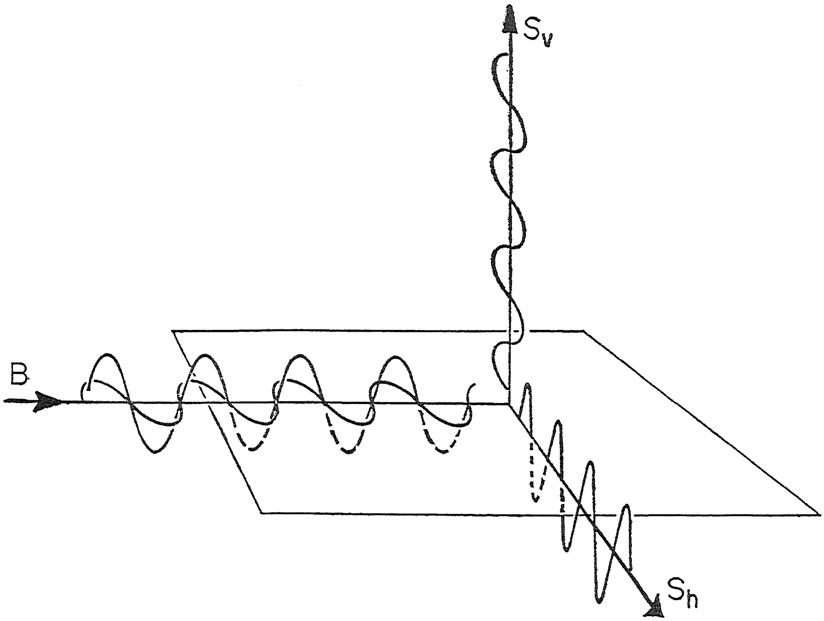


FIG. 6.4. Snapshot patterns of the rays S_h and S_v scattered at 90° from the incident, unpolarized beam B.

scattered at 45° was isolated by means of a pinhole diaphragm of lead, and was found to be highly polarized. Means of detecting and measuring the polarization have been described by McMaster and Hereford.

6.6. Polarization Produced by Slits and Gratings. If unpolarized light strikes a metal plate in which there is a long slender slit, the light that passes through the slit is partially linearly polarized. Thiessen (T-4) found that if the metal plate is relatively thick the electric vector of the transmitted light tends to be parallel to the slit and the degree of polarization is greater the narrower the slit and the longer the wavelength. The subject has been analyzed by Jones and Richards (J-34) also.

A transmission grating made from fine, closely spaced metallic wires may serve as a good polarizer for light of wavelength large relative to the spacing of the wires, according to du Bois and Rubens (D-20). Trentini (T-9) has conducted experiments involving two such gratings. Specific designs of wire-grating polarizers are described in the 1940 and 1942 patents 2,224,214 and 2,287,598 by C. H. Brown; the manu-

facturing process involves stretching wires embedded in a hot, transparent matrix such as glass until the diameter of the wire is reduced to less than 0.5 μ. In 1960 Bird and Parrish (B-27) produced wire gratings having extremely fine spacing, namely 0.5 μ. The polarizance of these gratings in the infrared range from 2 to 15 μ was very high and in the visual range was far from negligible. Constructing wire gratings suitable for use with, say, 3-cm radio waves is very simple; kits capable of appraising the polarization are manufactured by Central Scientific Company and Baird-Atomic, Inc.

Madden (M-7) has shown that ordinary reflection gratings, such as are used in visual-range spectroscopy, have polarizing properties. Meecham and Peters (M-16) have found that reflection echelette gratings designed for use with 3.2-cm waves exhibit polarizance that is in good accord with the predicted values.

7 RETARDERS AND CIRCULAR POLARIZERS

7.1. Introduction. Retarders, also called retardation plates, wave plates, and phase shifters, are *polarization-form converters.* Any polarization form can be converted to any other form by means of a suitable retarder. In principle the conversion is 100-percent efficient: there is no decrease in intensity and no increase in entropy flux. In practice the performance comes surprisingly close to this ideal. Retarders are used (in conjunction with linear polarizers) in the production of circularly and elliptically polarized light. They are used also in the analysis of such light; this is true, for example, in the technology known as photoelastic analysis (Sec. 10.8).

Various naturally occurring bodies act as retarders; by investigating the retardance of such a body, one may learn much about its structure. This is true of many kinds of crystals, various kinds of polymeric fibers and sheets, strained glass, and so forth.

It is clear, then, that the study of retarders is intimately tied up with the design and manufacture of certain types of polarizers, with the analysis of polarized light, and with the structural analysis of various transparent bodies. Consequently retarders deserve much attention in a book dealing with polarized light and polarizers.

7.2. Definition of a Retarder. A retarder may be defined as an optical element that, without appreciably altering the intensity or degree of polarization of a polarized monochromatic beam, resolves the beam into two components, retards the phase of one relative to the other, and reunites the two components, thus forming a single beam. An

equivalent definition is the following: a retarder is an optical element that conserves the polarization forms of incident beams having either of two particular polarization forms but alters the polarization forms of other types of beams. The two polarization forms that are conserved are characterized by the eigenvectors of the retarder. Depending on whether they are linear, circular, or elliptical, the retarder is called a linear, circular, or elliptical retarder. (Many authors refer to a circular retarder as an *optically active* device, or as a *rotator.*)

The eigenvector associated with the smaller refractive index (greater speed of propagation) may be called the fast eigenvector, and is the eigenvector that is emphasized in this book. The other eigenvector may be called the slow eigenvector. The two eigenvectors of any ideal homogeneous retarder are, of course, orthogonal.

Obviously, linear and circular retarders may be regarded as special cases of elliptical retarders.

A retarder that produces the same phase change irrespective of the wavelength of the light is called *achromatic.* Most retarders produce greater phase change for light of shorter wavelength (Sec. 7.7) and are called *chromatic.*

A retarder may consist of one layer or several separate layers. In some instances the separate layers are of one class and the combination acts as a retarder of a different class; such a combination may be called a *transcendent* retarder. For example, a certain stack of linear retarders may act as a circular retarder (Sec. 7.9); thus the combination is transcendent.

7.3. Optical Mechanisms Employed. Most retarders are of birefringence type or reflection type. A typical birefringence retarder consists of a single plate of quartz or calcite cut parallel to the optic axis, or a single plate of mica, a sheet of oriented cellophane, or a sheet of oriented polyvinyl alcohol. A stressed plate of glass is another example.

An aqueous solution of dextrose is a birefringence retarder, although in this instance *circular* birefringence is involved. A volume of air pervaded by a magnetic field parallel to the direction of propagation is another example of a birefringence circular retarder.

Reflection retarders rely on total internal (oblique) reflection. (Interesting effects are, of course, produced by reflecting a beam externally, obliquely, from a dielectric plate; but a plate used in this manner exhibits polarizance, and cannot be called a retarder.) The

Fresnel rhomb and Mooney rhomb, discussed in Sec. 7.7, are well-known reflection retarders.

Investigators working with microwaves or radio waves may employ retarders consisting of arrays of metallic wires, rods, or plates.

7.4. Definition of Retardance. The essential action of a retarder is to divide the incident beam into two orthogonally polarized components, retard one relative to the other, then recombine the two components to form a single emerging beam. The extent to which one component is retarded relative to the other is called the *retardance* δ (often called *retardation*). Note that retardance is a measure of the *relative* change in phase, not the absolute change; the birefringence of a typical retarder plate is small, and consequently the absolute change in phase caused by interposing the plate may be hundreds of times greater than the (relative) retardance. Note also that retardance is the *magnitude* of the relative change, and hence is always positive. Note, finally, that retardance is a constant of the body (assuming that we deal only with perpendicularly incident light of given wavelength) and hence is independent of the polarization form of the incident beam.

If the retardance is expressed in terms of cycles, the symbol δ_c may be used. If it is expressed in terms or degrees of phase angle, the symbol δ_{deg} may be used. Obviously $\delta_{\text{deg}} = 360\ \delta_c$. The most commonly used retarders are linear retarders having a retardance of 90° and 180°; these are often called quarter-wave plates and half-wave plates.

Sometimes it is convenient to speak of the *pathlength difference* Γ. This is the distance between corresponding wave fronts in the two emerging components, assuming that these are now traveling in vacuum. The distance is usually expressed in terms of millimicrons (mμ). If λ is the wavelength in vacuum, then $\Gamma = \lambda\delta_c$. The importance of the quantity Γ stems from the fact that the retardance δ_c of many typical bodies varies roughly as $1/\lambda$, and accordingly the pathlength difference Γ of such a body is approximately independent of wavelength.

Retardance is, of course, an *extensive* parameter of the retarder. The retardance of a birefringent plate that is mounted perpendicular to a beam of monochromatic light is the product of the planobirefringence J_p (defined in Sec. 5.2) and the ratio of thickness t to vacuum wavelength λ_v:

$$\delta_c = J_p(t/\lambda_v). \tag{7.1}$$

It should be noted that the path directions usually do not need to be considered. Usually the beam is normal to the plate, and consequently the two sets of wave fronts within the plate are parallel to the entrance face and each set has the same wave-normal direction; thus only one section of the refraction indicatrix needs to be considered and all the pertinent information on difference in speed is contained in J_p. The pathlength measured along the normal is the same for both components, and is simply the thickness t. In summary, the whole story of retardance is given by the quantities J_p, t, and λ_v. (If, however, the plate is tilted slightly, the two sets of wave fronts within the plate have slightly different wave-normal directions. Two different sections of the indicatrix must be considered. Furthermore, the lengths of the two wave-normal segments within the plate are different. Thus the computation of the retardance becomes more difficult in several respects.)

7.5. Performance Parameters of a Retarder. To describe a linear retarder one states the retardance δ and the azimuth ρ of the fast axis. (The definition of fast axis is presented in Sec. 5.2.) Azimuth may be defined with respect to a beam traveling horizontally along the positive Z-axis; the light source is assumed to be at the origin and the observer is assumed to be far out on the positive Z-axis looking toward the retarder and the light source. The azimuth of the retarder is the

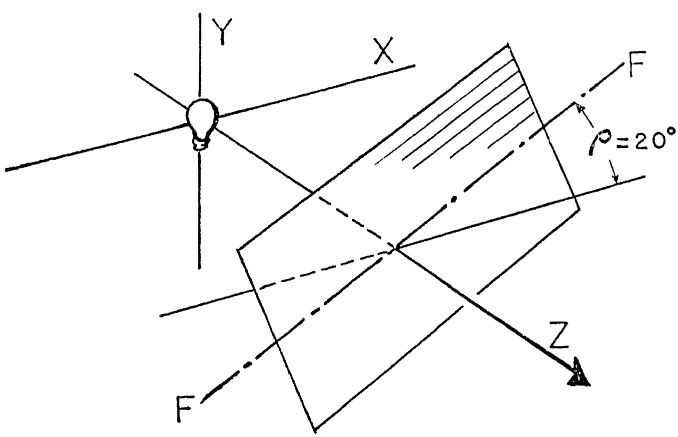


FIG. 7.1. Azimuth ρ (20° in this example) of the fast axis F of a linear retarder situated in a beam traveling horizontally along the positive Z-axis.

angle between fast axis and the X-axis, the angle being measured counterclockwise from the X-axis, as indicated in Fig. 7.1. (If the retardance is expressed as a phase angle, the investigator finds himself dealing with two kinds of angles: the azimuth angle ρ in real space and the phase angle δ in an abstract space. The temptation to confuse these angles must be resisted.) Alternatively, the azimuth may be indicated by means of the Poincaré-sphere representation of the fast eigenvector of the retarder. The pertinent point lies on the equator and, in the terminology of Chapter 2, is specified by an angle 2λ such that $2\lambda = 2\rho$. Note that the retardance *cannot be specified statically* on the sphere; rather, it governs the angular extent of the rotation to be employed, as explained in Sec. 7.6.

When using the Mueller calculus, one describes the linear retarder by means of a 4×4 matrix. The derivation of the matrices is given in Chapter 8, and a detailed list appears in Appendix 2. The matrices bear a resemblance to the rotator matrices commonly used in transformations among nonparallel sets of cartesian coordinates. The resemblance is to be expected in view of the relation (Chapter 2) between the Stokes parameters and the Poincaré-sphere representation. The rotation matrices are the algebraic counterparts of the geometric operations of rotating the sphere.

Instead of specifying the Mueller matrix, one may specify (*a*) the fast eigenvector in terms of its Stokes vector and (*b*) the retardance. However, specification in terms of the Mueller matrix usually is more fruitful, as explained in Chapter 8.

When using the Jones calculus, one describes the retarder by means of a 2×2 matrix; see Chapter 8 and Appendix 2. If the retarder is of homogeneous type one may specify (*a*) the fast eigenvector in terms of the Jones vector and (*b*) the retardance.

To specify a circular retarder, one indicates the handedness and the retardance. In this book the handedness is taken as that of the fast eigenvector; thus the handedness is right if right-circularly polarized light is governed by the smaller index and hence is propagated at greater speed. It is easily shown that this definition implies that a *right*-circular retarder is one that causes the vibration direction of a linearly polarized beam to be turned *clockwise* as judged by an observer situated far out along the Z-axis and looking toward the retarder and the lamp beyond; the vibration direction is turned clockwise by a

rotation angle ζ that is equal to *half* the retardance; that is, $|\zeta| = \frac{1}{2}\delta$. Alternatively, one may describe the retarder in terms of its Mueller matrix or Jones matrix.

To describe an elliptical retarder, one may state the retardance and the fast eigenvector. The eigenvector may be specified in terms of ellipticity, handedness, and azimuth ρ, or it may be specified in terms of the Poincaré sphere, the Stokes vector, or the Jones vector. Matrices, also, may be used.

7.6. Predicting the Effect of a Retarder. The effect of a retarder on a beam of linearly polarized light can be predicted by naïve graphical means, by recourse to the Poincaré sphere, or by use of the Mueller calculus or the Jones calculus. The naïve graphical method, although usually quite cumbersome and *not* to be recommended, deserves brief mention because of its historical importance and its direct and obvious relation to the electromagnetic theory.

To find, by graphical means, the effect of a linear retarder on a linearly polarized beam, one considers the electric vector P_1 that characterizes the beam at some arbitrary instant t_1 and at the location where the beam enters the retarder, then determines the corresponding vector P_1' of the beam emerging from the retarder. One repeats the process for various other instants t_2, t_3, . . . until enough vectors of the emerging beam have been found that one can draw a smooth curve through the tips of the vectors and thus obtain the ellipse that represents the polarization form of the emerging beam.

Figure 7.2 illustrates the application of the method to a linear retarder having a retardance of 45° and a fast axis at 20° to the horizontal. The incident beam is assumed to be horizontally polarized. The figure shows how, using ruler, compass, and protractor, one finds the point P' of the emerging beam's (elliptical) sectional pattern that corresponds to point P of the incident beam's (linear) pattern. The arbitrary instant is assumed to be such that P corresponds to the maximum displacement to the right. One starts by projecting OP onto the fast and slow axes, to obtain OU and OV respectively. For simplicity, one considers OU, the projection on the fast axis, as being transmitted without phase change, and finds the extent to which OV, the projection on the slow axis, is changed by the relative retardance δ. This is done by drawing a circle with radius OV, drawing a radial line r at an angle equal to the retardance ($\delta = 45°$), and projecting r

onto the slow axis to obtain OV'. By drawing the resultant of OU and OV' one arrives at OP'. The point P' is one point on the ellipse. Because of the symmetry that the ellipse must exhibit, a corresponding point P'' may be marked without further computation.

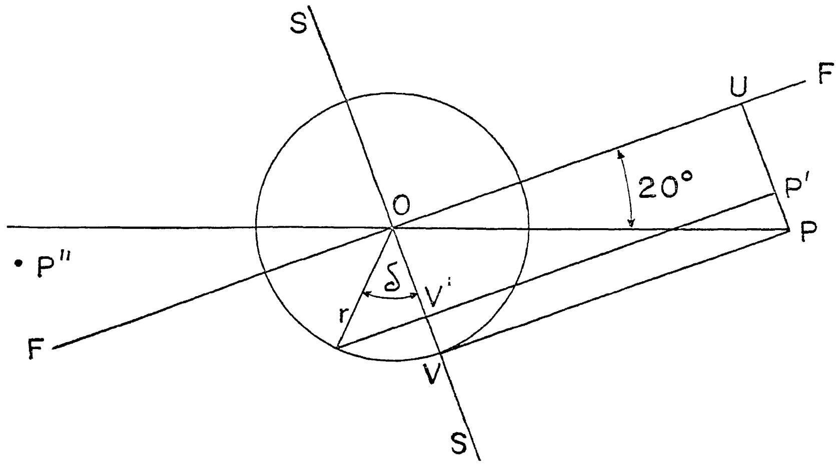


FIG. 7.2. Diagram used in finding the effect of a 45° linear retarder at 20° on a beam of horizontally linearly polarized light; FF and SS denote the fast and slow axes of the retarder; P represents the instantaneous vector of the incident beam and P' represents the corresponding vector of the emerging beam.

A second pair of points on the ellipse may be found by assuming that, in the original projection of OP, the component on the slow axis is unchanged in phase and the component on the *fast* axis is altered (by the angle 45°). One constructs a circle with OU as radius, draws a radius vector rotated 45° from the direction of the segment OU, projects this onto the fast axis, and again constructs the pertinent resultant.

A third pair of points may be found by employing the original projections OU and OV, constructing circles corresponding to each, constructing a radius vector for each, rotating these through angles A and $A + \delta$, projecting these onto the respective axes, and finding the resultants of these projections. For each arbitrary choice of A (for example, 90°, $-90°$, 45°, $-45°$), one arrives at a new point on the ellipse, or a new *pair* of points if advantage is taken of symmetry. Three or four pairs of points may suffice, if the investigator wishes to find merely the approximate shape and azimuth of the ellipse.

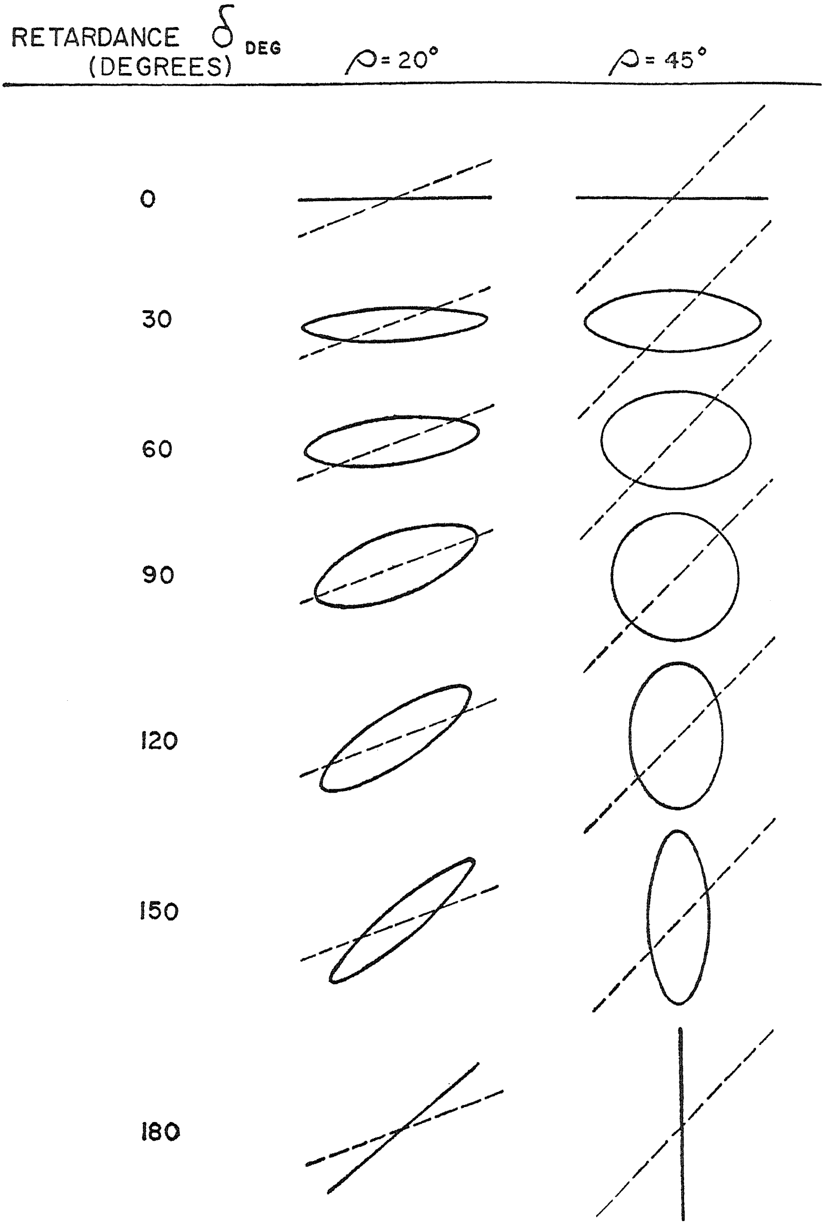


FIG. 7.3. Elliptical polarization forms that result when a linear retarder is inserted in a beam of horizontally polarized light. The broken line indicates the fast axis of the retarder.

Figure 7.3 shows the variety of ellipses that result from inserting a linear retarder in a beam of horizontally polarized light, assuming that the fast axis of the retarder is at $\rho = 20°$ and the retardance δ_{deg} has a variety of values ranging from 0° to 180°. Obviously, the ellipticity is greatest when $\delta_{\text{deg}} = 90°$. The azimuth of the major axis of the ellipse is a maximum when $\delta_{\text{deg}} = 180°$ (it is then equal to twice 20°, or 40°). All of the ellipses are right-handed, but become left-handed if the retarder is turned so that the slow axis has the position formerly occupied by the fast axis.

Figure 7.4 shows the ellipses that result when the retardance is held

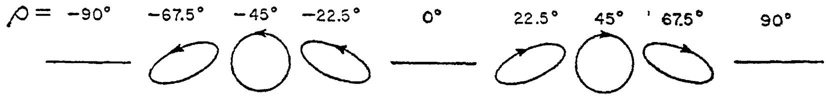


FIG. 7.4. Elliptical polarization forms that result when a 90° linear retarder with fast axis at various azimuths ρ is inserted in a horizontally polarized beam.

constant at 90° and the azimuth ρ of the fast axis has various values from −90° to +90°. The resulting figure is a circle when $\rho = \pm 45°$; the handedness is right when $90° > \rho > 0$ and left when $-90° < \rho < 0$.

When the incident beam is elliptically polarized, and especially when an elliptical retarder is included, the use of the naïve graphical method is to be discouraged. Rather, the more modern methods, discussed in the following sections, should be used.

Poincaré-Sphere Method. The preferred method of predicting the effect of a retarder on a beam of polarized light is to employ the Poincaré sphere; see Chapter 2, and also a detailed explanation by Ramachandran and Ramaseshan (R-28). The method is simplicity itself. One marks the point P that describes the polarization form of the incident beam, and rotates the sphere about the appropriate axis and through the appropriate angle. The new location of point P (relative to the original sphere) characterizes the polarization of the emerging beam.

As axis of rotation one employs the radius vector from the center of the sphere to the point R that describes the fast eigenvector of the retarder. One rotates the sphere through an angle equal to the retardance δ_{deg} (which has been defined so as to be always positive). The sense of rotation is always the sense that appears clockwise to an observer situated far out on the (extended) radius vector.

The rotation may be carried out on an actual sphere, or on graph paper suited to stereographic projection. Suitable spheres, provided with degree scales, have been described by Jerrard (J-14) and Koester (K-12); special graph paper of "Wulff-net" type has been described by Hartshorne and Stuart (H-16) and employed successfully by Koester (K-12) (such paper is obtainable from the University of Toronto Press). A free-hand perspective sketch of the sphere (Fig. 7.5) will often suffice for qualitative purposes.

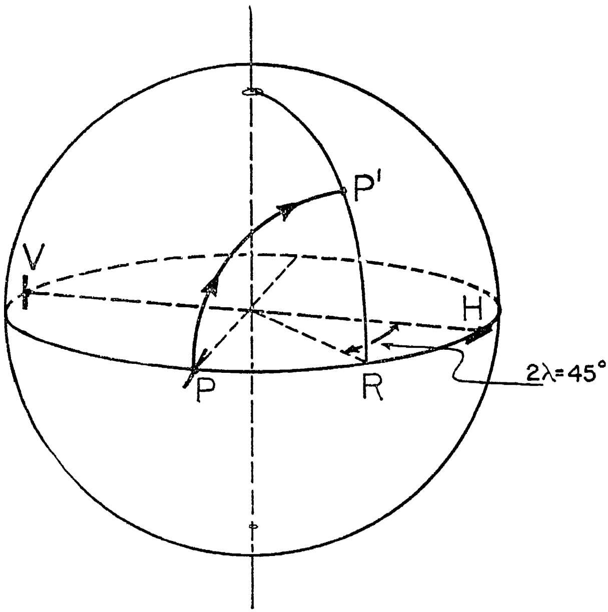


FIG. 7.5. Use of the Poincaré sphere in predicting the effect of a 90° linear retarder R (with fast axis at 22.5°) on a linearly polarized beam P (with vibration direction at 45°).

As an illustration of the method, consider a beam of light that is linearly polarized at 45°, as suggested by point P in Fig. 7.5, and suppose that the beam encounters a linear retarder whose retardance is 90° and whose fast axis is at 22.5° as indicated by R. Then the pertinent rotation is indicated by the right-handed quadrantal arc that starts at P and is generated by a rotation about the radius vector through R. The end point P' is the answer: it indicates the polarization form of the emerging beam. Clearly the beam is left-elliptically polar-

ized (because P' lies in the upper hemisphere); the major axis is at 22.5° (because P' lies on the same meridian as R, namely the meridian for which $2\lambda = 45°$); the ellipticity is $\tan 22.5° = 0.4142$ (because $2\omega = -45°$, and hence $|\omega| = 22.5°$).

If the linearly polarized incident beam were to have a vibration direction that differs by 45° from the 90° retarder's fast axis, the end point of the arc would necessarily be at a pole; hence the use of 90° retarders in converting linearly polarized light to circularly polarized light.

If, in Fig. 7.5, the retardance were 180°, the arc would be a semicircle and the end point would be found at H, representing horizontally polarized light. If the arc were extended to form a complete circle, the various points on the arc would represent every possible outcome from inserting (in the given beam) a linear retarder having every possible value of retardance (and having the given azimuth of fast axis).

If the retarder is of right-circular type, the point R coincides with the lower pole of the sphere and the rotation is performed about the radius vector passing through that pole. The resulting arcs are all parallel to the equator, showing that if the incident beam is right-elliptically polarized the emerging beam will also be right-elliptically polarized and with the same ellipticity. Changing the retardance of a circular retarder changes only the *azimuth* of the major semiaxis of the emerging beam.

In general, the Poincaré-sphere representation is useful not merely in *carrying out* a problem involving polarized light and retarders but also in *formulating* the problem — thinking about it and talking about it. Various puzzling problems become clear once the investigator translates the problem into the graphic, maximum-brevity language of the Poincaré sphere.

In general, any polarization form P_1 can be converted to any other form P_2 by finding (on the sphere) a point R midway between them, then inserting in the beam P_1 a 180° retarder whose fast eigenvector corresponds to R. If both P_1 and P_2 indicate *linear* (and nonorthogonal) forms, the required linear retarder is a 180° retarder and its fast axis must bisect the angle between the vibration directions implied by P_1 and P_2. (Whether the fast axis or the slow axis bisects the angle is immaterial; any two points P_1 and P_2 are, of course, connected by *two* arcs of a great circle — a short arc and a long arc; the midpoint of

either represents a solution for R, and these two solutions correspond to having the fast axis or the slow axis bisect the angle in question. Stated differently: the fast axis of the retarder may bisect either the acute or the obtuse angle between P_1 and P_2.)

If a train of, say, three retarders is used, three rotations must be performed in succession, about the radius vectors through the three points R_1, R_2, and R_3 (Fig. 7.6). The rotations are described by three

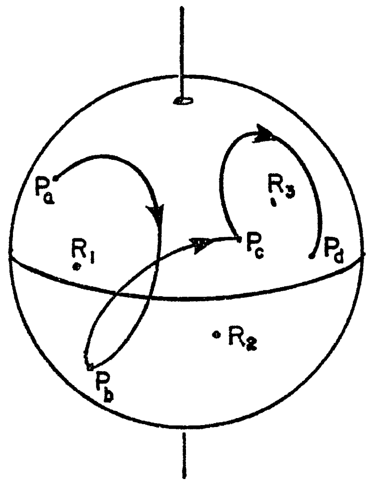


FIG. 7.6. Use of the Poincaré sphere in finding the result of interposing a train of three elliptical retarders in a beam of elliptically polarized light.

arcs, each connected to the next. *The properties of the finally emerging beam P_d are determined without need for pausing and interpreting the intermediate outcomes P_b and P_c.*

Other Methods. The Mueller calculus may be used to advantage in many instances and the same is true of the Jones calculus. The procedure is to write down the vector representation of the incident beam, write down the matrix of the retarder, and multiply the vector by the matrix. If, say, a train of three retarders is involved, three matrices are required, and three multiplications are needed. The procedures are explained in detail in Chapter 8.

Partially Polarized Light. So far we have been considering the action of a retarder on a beam of completely polarized light. Happily, no other type needs any detailed consideration. A retarder has no significant effect on unpolarized light; and since any partially polar-

ized beam can be resolved (mentally) into a completely polarized beam and an unpolarized beam, one merely treats the former by any of the methods discussed above and regards the latter as being unchanged.

7.7. Chromatic Linear Retarders. As examples of chromatic retarders, thin plates of calcite, mica, or quartz may be mentioned. The planobirefringence J_p of a section of calcite cut parallel to the optic axis is 0.172 for sodium light; accordingly, the thickness of a 90° retarder of calcite is 870 mμ, by Eq. (7.1). To fabricate such thin plates is very difficult. The fabrication is simplified by employing a material having smaller planobirefringence. Mica, in the form of fluophlogopite, muscovite, or biotite, performs excellently, and thin sections are easily produced by cleavage. The cleavage operation is made more manageable if the mica is immersed in water and is cleaved slowly. The visibility of the operation is improved if the mica is situated between crossed polarizers; detailed procedures are described by Jerrard (J-13) and Dobrowolski (D-13). Nearly any desired thickness can be achieved by trial and error; the typical planobirefringence is 0.004, and accordingly a section 50 μ thick provides approximately 90° retardance for sodium light.

Quartz, also, is used in high-quality linear retarders. The section is cut parallel to the optic axis (otherwise elliptical or circular birefringence is exhibited). The planobirefringence of such a section is about 0.009, and accordingly a section about 15 μ thick provides (for sodium light) a retardance of about 90°.

Besides crystals, sheets of oriented organic polymers may be used. A sheet of polyvinyl alcohol, if warmed and unidirectionally stretched, constitutes an excellent retarder. The method of preparation has been described in papers by Land and West (L-8) and by West and Makas (W-20), in various patents by Land, and in West's patent 2,441,049. Ordinarily the value of over-all birefringence achieved is somewhat less than 0.01, but in extreme cases (axial ratio exceeding 5) values of about 0.03 or even 0.034 have been realized (L-8); the two indices are then about 1.560 and 1.526. The planobirefringence is of parallel type; that is, the slow axis is parallel to the stretch direction. In many of the commercially produced retarders that consist of polyvinyl alcohol, a modest value of planobirefringence is employed; the thickness of a 90° retarder is of the order of 20 μ. Section 7.10 discusses the

application of such retarders to the manufacture of circular polarizers.

Mylar sheets, as produced commercially, exhibit some planobirefringence; but if a slender strip of the material is warmed and stretched, a planobirefringence as great as 0.2 may be achieved. Such a material is well suited to the production of retarders having, say, 10 or 20 cycles of retardance and hence useful in pseudodepolarizers (discussed in Sec. 7.12). Oriented nylon sheet, also, may be employed; such material has recently become commercially available.

Cellulose acetate butyrate, if stretched, exhibits planobirefringence. The relation between planobirefringence, directions of the principal axes of refraction, temperature of the specimen during the stretching operation, and time rate of stretching has been investigated by McNally and Sheppard (M-6) and by West and Makas (W-20), and a number of anomalies have been found. The effect of plasticizer has been investigated also. Curiously enough, certain stretched sheets may exhibit no planobirefringence near 550 mμ but much planobirefringence near 400 and 700 mμ, and the class of the planobirefringence (that is, whether of parallel or perpendicular type) may be opposite at these two wavelengths.

Stretched sheets of cellulose nitrate, studied by West and Makas (W-20), have unusual properties. If the extent of stretching is such that the axial ratio is about 2 or 2.5, the retardance δ_c may be *greater* for 700-mμ light than for 400-mμ light. Accordingly, this material has been found useful in the production of achromatic retarders, by

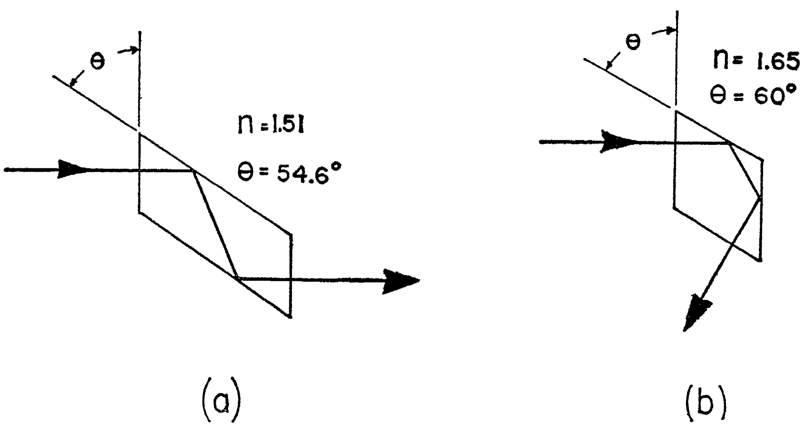


FIG. 7.7. Designs of (*a*) the Fresnel rhomb ($n = 1.51, \theta = 54.6°$) and (*b*) the Mooney rhomb ($n = 1.65, \theta = 60°$).

compensating the chromatism of a more typical material (see Sec. 7.8).

Many other polymeric materials may, of course, be used as retarders. Commercially produced cellophane often has a retardance (Fahy, F-2) of 90° to 180° and the same applies to "Scotch" tape.

The Fresnel rhomb (Fig. 7.7a) is a specially-shaped rhomb of glass that totally internally reflects the beam twice, and achieves a retardance of 90°. If the index is 1.51, the angle of each apex of the rhomb should be approximately 54.6° (J-9); the exact optimum choice of angle must be determined by trial and error, according to Ditchburn (D-10), if small and unpredictable phase changes at the entrance and exit surfaces are to be taken into account. If the incident beam strikes the rhomb obliquely, instead of along the normal, a different value of retardance results.

A rhomb described in 1952 by Mooney (M-24) has an index of 1.650 and employs 60° angles (Fig. 7.7b). It has the disadvantage of producing an emerging beam that is aimed obliquely backward, but the advantage of much smaller obliquity effects, so that beams of greater angular spread may be used.

Retarders suitable for use in the ultraviolet range have been discussed by McDermott and Novick (M-1); polyvinyl alcohol films, thin sheets of mica, and thin plates of quartz may be used.

7.8. Achromatic Linear Retarders. Whether a retarder is chromatic or achromatic is of little consequence in applications where the light employed is monochromatic and the retarder has been designed for the particular wavelength in question. However, a retarder to be used with white light should be of achromatic type; otherwise the retardance δ_{deg} will be different for different wavelengths and the outcome of the operation will be complicated. (In some instances, and in the hands of an experienced investigator, advantage may be taken of the retarder's chromatism; see Chapters 9 and 10.)

The Fresnel rhomb is nearly achromatic, but has obvious drawbacks as regards bulk, deviation, and cost. Accordingly, efforts have been made to develop sheet-type achromatic retarders. Some of these efforts have been described by West and Makas (W-20); see also West's patent 2,441,049. A fair degree of success may be achieved by using a series combination of a stretched sheet of cellulose acetate and a stretched sheet of cellulose nitrate; the two sheets are mounted with their respective fast axes crossed, so that the normal type of chro-

matism of the cellulose acetate is countered by the abnormal type of chromatism of the cellulose nitrate. However, such a combination presents questions as to stability, and the cellulose nitrate may constitute a fire hazard.

In 1955 Pancharatnam (P-1) proposed a novel kind of achromatic 90° retarder. It consists of three layers, all of the same material — mica, for example. Layers 1 and 2 have 180° retardance for 589-mμ light; layer 3 has 90° retardance. The slow axes of the three layers make angles of 6°52′, 34°32′, and 100°21′ with some reference direction (the direction of vibration of an incident beam of linearly polarized light that is to be converted to circularly polarized light). Such a system is almost perfectly achromatic from 400 to 780 mμ.

Later Pancharatnam proposed a slightly different design (P-2). Layers 1 and 3 have 115°42′ retardance and their slow axes are parallel. Layer 2, situated between them and oriented at 69°54′, has a retardance of 180°. The combination is nearly achromatic from 410 to 680 mμ.

In 1959 Koester (K-12) described some slightly different designs of achromatic retarders of multilayer type.

Lostis (L-26) has employed a nearly achromatic 180° retarder that entails total internal reflection from an oblique, specially coated surface. The variation in retardance throughout the visual range does not exceed 4 percent.

7.9. Circular Retarders. A basal section of quartz is a typical example of a circular retarder. Whereas a section cut parallel to the optic axis is a linear retarder, a section cut perpendicular to this axis is a circular retarder; an oblique section is an elliptical retarder. The variation of retardance with wavelength has been discussed by Hurlbut and Rosenfeld (H-39); see also C-9, and *American Institute of Physics Handbook*, Sec. 6, p. 98. Depending on the type of quartz crystal employed, the handedness of the retarder may be right or left. Many other crystals also, when suitably sectioned, exhibit circular retardance or elliptical retardance.

Various pure liquids, such as turpentine, exhibit circular retardance. The same is true of various liquid solutions, such as an aqueous solution of dextrose (Sec. 10.5). Even a pure gas may exhibit circular birefringence, if the gas is pervaded by a magnetic field (Sec. 10.7).

The combination of two 180° linear retarders oriented with fast

axes at 45° to one another acts as a 90° circular retarder. This may be demonstrated experimentally, or may be proved by multiplying the appropriate Mueller or Jones matrices of the two linear retarders and observing that the product is identical to the matrix of a 90° circular retarder (Sec. 8.4). The combination in question is called a transcendent retarder. Jones has shown that a skewed series of n linear retarders is equivalent to a circular retarder, and experimental confirmation of this has been provided by Dawson and Young (D-3).

7.10. Use of Retarders in Producing Circularly Polarized Light. Conceivably, one could make a practical (circular) polarizer out of a material that exhibits circular dichroism; see Bruhat (B-53, B-55) and Cotton (C-29). An easier method, however, is to use a series combination of a linear polarizer whose transmission axis is at some azimuth θ and a 90° linear retarder whose fast axis is at azimuth $\theta \pm 45°$. As explained in Sec. 7.6, such a combination produces circularly polarized light.

Many investigators who require circularly polarized light arrange their own combination of linear polarizer and linear retarder. The polarizer may be of birefringence, dichroic, or other type, and the retarder may, for example, be of mica. However, the commonest type of circular polarizer is the CP-HN polarizer (see Land's patents 2,018,963 and 2,099,694), which consists of a sheet of HN-35 (or other HN polarizer) and a sheet of stretched polyvinyl alcohol. The latter has been stretched so as to have a retardance of approximately 90° for 550 or 560 mμ; it is laminated to the linear polarizer at an angle of 45°. If desired, the combination may be laminated between protective covers of organic polymeric material or glass.

Such a polarizer may be of either handedness, depending on whether the retarder is oriented at +45° or −45°. For reasons relating to convenience of manufacture, a majority of the plastic-laminated CP-HN polarizers are right handed and a majority of the glass-laminated polarizers are left handed. (One could, of course, make an "ambidextrous circular polarizer" by sandwiching a linear polarizer at 45° between two suitably oriented 90° linear retarders.)

The CP-HN-35 polarizer has a total luminous transmittance k_v of approximately 35 percent. As implied by Fig. 7.8, the spectral k_t curve is nearly horizontal, showing that the polarizer is nearly neutral in color. The round-trip nominal total transmittance k_{ntrt}, defined in

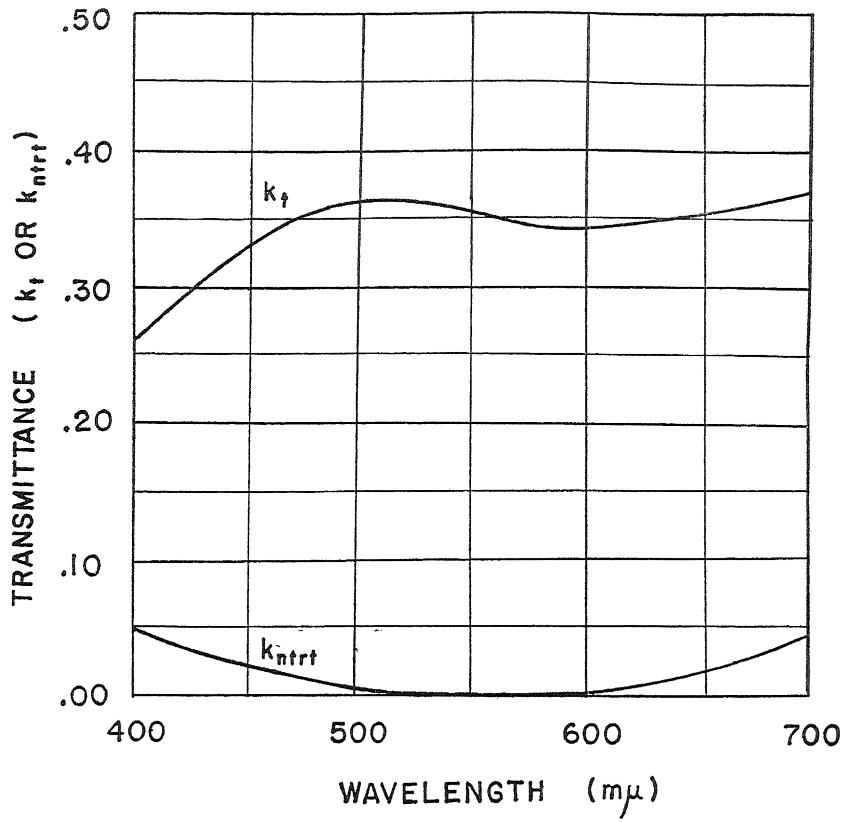


FIG. 7.8. Total transmittance k_t and round-trip nominal total transmittance k_{ntrt} of a CP-HN-35 polarizer.

Sec. 3.6, is shown in the same figure; it has a value very close to zero near the middle of the visual range, where the retardance differs but little from 90°. (Chapter 8 presents the matrices of an ideal two-layer circular polarizer.)

7.11. Use of Retarders in the Analysis of Elliptically Polarized Light. The systematic analysis of polarization type, form, and degree has been considered in several textbooks, including those by Ditchburn (D-10, p. 374) and Jenkins and White (J-9, p. 540). Consequently it does not need to be considered here. This is particularly true since an investigator seldom encounters situations in which both the type and the degree of polarization are unknown.

The commonest situation is that in which the beam is known to be linearly polarized, and the vibration azimuth remains to be determined. The determination is very simple: one inserts in the beam, as analyzer, a linear polarizer whose transmission axis is known, and finds the azimuth A which produces the best extinction. The vibration azimuth of the original beam is then simply ($A \pm 90°$). Precise measurements may be made with the aid of split-field analyzers (see Fig. 7.9) or by recourse to polarimeters, such as those discussed in Sec.

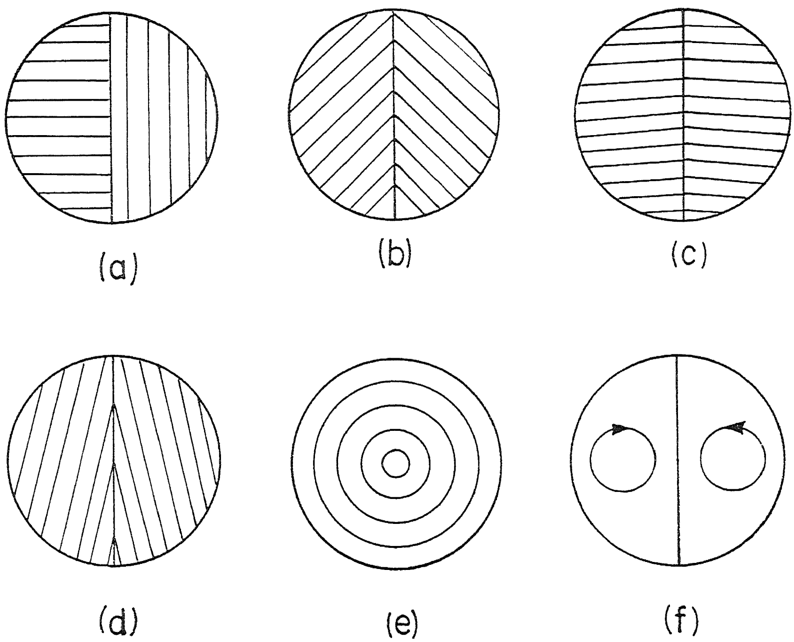


FIG. 7.9. Split-field analyzers: (*a*) 0° − 90° analyzer; (*b*) 45° − 45° analyzer (Bravais dichroscope); (*c*) 3° − 3° analyzer; (*d*) 87° − 87° analyzer; (*e*) concentric-circle analyzer; (*f*) right-left analyzer (circular dichroscope). The hatch marks indicate the transmission axis of the region in question.

10.5. In some instruments one half of the split field is provided by an obliquely mounted polarizing prism (Lippich prism) that covers half of the visual field; the prism is tilted slightly so as to produce a small change in azimuth of polarization here. The optimum tilt has been determined by Rudolph (R-16), Koester (K-13), and Inoué and Koester (I-7).

Another common problem is the determination of the degree of polarization V of a beam that is known to be partially *linearly* polarized. Here one employs as analyzer a linear polarizer whose k_2 values at all pertinent wavelengths are essentially zero. One inserts this analyzer in the beam and turns it to the azimuths giving the *most* intense and *least* intense transmitted beam. One measures these two intensities, I_d and I_i (Sec. 1.5), and then computes V directly by means of the familiar equation

$$V = \frac{I_d - I_i}{I_d + I_i}.$$

A photoelectric method of measuring degree of polarization (and of measuring the extent to which gases and liquids depolarize light) has been described by Weill (W-14); his apparatus includes Wollaston and Glazebrook polarizers.

The determination of the handedness of light that is known to be circularly polarized is simple if one possesses a right-circular polarizer: one inserts this polarizer in the beam, as an analyzer; if it extinguishes the beam, the beam is left-circularly polarized; if it leaves the beam almost unchanged in intensity, the beam is right-circularly polarized. Another approach is to use a 90° linear retarder whose fast axis is known; one inserts the retarder in the beam, with the fast axis horizontal; if the azimuth of the resulting (linearly polarized) beam is *plus* 45°, the original beam's handedness is *right*. Some persons are able to distinguish right- and left-circularly polarized light directly by eye, as is explained in Sec. 10.2.

A more difficult problem is the determination of the constants of an elliptically polarized beam (Sec. 1.2). For this purpose one may employ a simple *ellipsometer*, such as that of Brown (B-51) or Skinner (S-17), or various more elaborate ellipsometers such as those manufactured by Gaertner Scientific Corporation or Quantum, Inc.; see also Rothen (R-14) and Rudolph (R-16). The heart of any ellipsometer is a calibrated retarder, or compensator, which usually consists of a linear retarder that can be adjusted so as to have the desired retardance (say 90°) for whatever wavelength is concerned. At least eight kinds of retarders are available; their merits have been compared by Jerrard (J-11), Jessop (J-16), Rich (R-5), Richartz and Hsu (R-7), and Walker (W-2). Detailed accounts of the Senarmont compensator are available (G-1, H-16, J-11), and the same applies to the Babinet

compensator (D-10, H-13, P-30), the Tardy compensator (J-16), the Berek compensator (H-16), and various other compensators. The standardization of compensators and other retarders has been considered by Laine (L-2) and Randall (R-3). The appraisal of beams having very small ellipticity has been discussed by Tsurumi (T-10). The Savart plate and other auxiliary analytical tools have been described by Wood (W-26), Strong (S-31), Filon (F-9), Johannsen (J-18), Ramachandran and Ramaseshan (R-2), Van Heel (V-3), and many others.

Methods of appraising the polarization form of a microwave beam have been reviewed by Allen (A-3).

7.12. Use of Retarders as Pseudodepolarizers. In certain instances the (slight) polarization exhibited by typical beams of light can be a serious nuisance. This is true of the light employed in spectrophotometers that are used for measuring the total transmittance or reflectance of specimens that exhibit polarizing tendencies. The polarizing tendencies of the dispersing prisms used in spectrophotometers have been discussed by Bolla (B-39), Charney (C-13), Ellis and Glatt (E-14), Hyde (H-43), and Baxter *et al.* (B-8). The elimination of polarization in a beam is surprisingly difficult; indeed, no simple and fully satisfactory method of depolarizing light is known. True depolarization necessarily involves a decrease in temperature of the dominant component and an increase in entropy flux of the beam as a whole (Sec. 2.6). Some success can be achieved by scattering the light from a cloud of randomly arranged particles of appropriate size and shape, or by employing an integrating sphere; however, such schemes drastically alter the direction and angular width of the beam, and tend to be very wasteful of light.

A better approach is that which involves *pseudodepolarization.* Here one does not attempt to achieve truly unpolarized light (Secs. 1.4 and 2.3); one attempts instead to produce such a great variety, or mixture, of polarization forms that the over-all effect is the same, for practical purposes, as if the light were indeed depolarized. One procedure is to interpose a series of linear retarders that are rotating rapidly and with unrelated speeds; the emerging beam may then be difficult to distinguish from unpolarized light. A simpler procedure, applicable when a wide range of wavelengths is involved, is to use a thin wedge of quartz or other *chromatic* linearly retarding material, as discussed

by Hughes (H-35). Alternatively, one may use a fixed *train* of chromatic linear retarders that have very large values of retardance and different directions of fast axis. For example, one may use two chromatic linear retarders that have, say, 15 cycles of retardance for 550-mμ light and are positioned with fast axes at 45° to one another. For most types of polarized incident beam, the emerging beam will be found to have a great variety of polarization forms; the polarization form varies cyclically with wavelength, and passes through a great many cycles throughout the interval from 400 to 700 mμ. Billings (B-23) has investigated the behavior of such pairs of retarders and has shown that the emerging beam's resemblance to unpolarized light may be made very close if the design of the pseudodepolarizer is tailored to the particular form of polarization of the incident beam. Oriented sheets of mylar (Sec. 7.7) serve excellently as multicycle chromatic retarders.

8 MUELLER CALCULUS AND JONES CALCULUS

8.1. Introduction. The Mueller calculus and the Jones calculus are two new tools that are eminently useful in predicting the result of interposing several polarizers and retarders in a beam of light; in the Mueller calculus the effect of scatterers can also be included. The various matrices that constitute the building blocks of the new calculi are tabulated in Appendix 2, and an investigator who wishes to employ one of these calculi should refer to that appendix. The present chapter explains how the matrices are derived and how they are used. (In preparing this chapter the author was fortunate in receiving extensive assistance from Dr. R. Clark Jones, inventor of the Jones calculus.)

8.2. Outline of the Mueller Calculus. The Mueller calculus is a matrix-algebraic method of specifying a beam of light and the optical devices encountered by the beam, and computing the outcome.

One may ask: Why is a special kind of calculus needed? What is wrong with the conventional algebraic methods and the conventional trigonometric methods? Actually there is little wrong with them except that they become extremely cumbersome when the number of polarizers or retarders is large; the arithmetic required is voluminous, and the procedure is intricate — and different for each different kind of problem. Also, one occasionally encounters a problem where they are not applicable.

Why are the conventional methods cumbersome? Because of the complicated nature of light and matter. Even a single ray of monochromatic light requires several parameters for a full specification: to

state the ray's amplitude or power is not enough; the degree of polarization (one parameter) and the polarization form (two or three additional parameters) must be stated also. Similarly, a polarizer does not have just *one* characteristic transmittance but *two.* Retarders and scatterers, too, involve several parameters.

The Mueller calculus takes advantage of the discovery that it is possible to (*a*) condense all the necessary parameters for describing a beam of light *into a single package,* (*b*) condense all the necessary parameters for describing a given polarizer, retarder, or scatterer *into a single package,* and (*c*) provide a set of rules whereby the result of interposing any given polarizers, retarders, or scatterers in a given beam *can be determined merely by multiplying the appropriate packages together in standard manner.* Thus the outcome of any pertinent experiment can be determined by one fixed procedure: selecting the appropriate packages from a table and multiplying them together.

The package describing the light beam is simply the four-parameter *Stokes vector,* described in Chapter 2. As explained there, the four parameters I, M, C, S are related to intensity, preference for horizontal polarization, preference for plus 45° polarization, and preference for right-circular polarization. The vector (a column vector) is written vertically or (less formally) horizontally:

$$\begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} \quad \text{or} \quad \{I, M, C, S\}.$$

The wavelength range (bandwidth) of the light is assumed to be *broad* enough that the light may, for example, be unpolarized, and *narrow* enough that the optical devices in question may be regarded as achromatic within this range.

The package describing the polarizer, retarder, scatterer, or other optical device is called a *Mueller matrix.* It is a 4×4 matrix, and thus contains 16 elements. Happily, most of the elements are zero for various ideal devices. The individual matrix is indicative not only of the composition of the device but also of its orientation (azimuth); thus the matrix of a linear polarizer whose transmission axis is horizontal is different from the matrix of a similar polarizer that has been turned so that its axis is at, say, 37°. If one turns an optical device around, so that a different face serves as entrance face, a different

matrix may be required. Also, tilting the device so that the light is incident obliquely may require use of a different matrix.

The matrix describes the optical device with respect to *one* emerging beam; two beams emerge from a Wollaston prism, but a Mueller matrix can serve with respect to one of these only; if both emerging beams are of interest, two matrices must be used and two separate calculations must be performed.

The main rules used in performing the multiplications are the standard rules of matrix algebra. One must also observe the following convention: the vector representing the incident beam must be written at the right, and the successive matrices representing the successively encountered devices must be arranged in order, the matrix of the last-to-be-encountered device being written at the left.

8.3. Examples of Applications of the Mueller Calculus. Consider an experiment in which a unit-intensity beam of unpolarized light strikes the simplest kind of ideal, linear polarizer, whose transmission axis is horizontal. The Stokes vector of the incident beam is $\{1, 0, 0, 0\}$ (Sec. 2.3). The Mueller matrix of the polarizer is

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

To find the properties of the emerging beam, one multiplies the vector describing the incident beam by the matrix describing the polarizer. One writes:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This has the general form

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix},$$

and, according to the usual rules of matrix multiplication, the product has the form

$$\begin{bmatrix} m_{11}I + m_{12}M + m_{13}C + m_{14}S \\ m_{21}I + m_{22}M + m_{23}C + m_{24}S \\ m_{31}I + m_{32}M + m_{33}C + m_{34}S \\ m_{41}I + m_{42}M + m_{43}C + m_{44}S \end{bmatrix}.$$

It is to be noted that each row of the product contains terms that are connected by plus signs. Accordingly each row consists, essentially, of a single element. Thus the expression is a column vector, not a 4×4 matrix.

The general form of the product appears formidable, but in practice it is often very simple, because so many of the elements are zero. In the present example the product is

$$\begin{bmatrix} \frac{1}{2}(1) + \frac{1}{2}(0) + 0(0) + 0(0) \\ \frac{1}{2}(1) + \frac{1}{2}(0) + 0(0) + 0(0) \\ 0(0) + 0(0) + 0(0) + 0(0) \\ 0(0) + 0(0) + 0(0) + 0(0) \end{bmatrix} \text{ or simply } \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}.$$

The last expression is the answer. The first element is $\frac{1}{2}$, indicating that the emerging beam has an intensity of $\frac{1}{2}$. The second term is positive, indicating a preference for horizontal polarization. The last two terms are zero. Thus one concludes, in view of Chapter 2, that the resulting beam is 100-percent linearly, *horizontally*, polarized.

As a second example, consider what happens when a unit-intensity horizontally polarized beam strikes a simple, ideal, 180° retarder whose fast axis is at 45°. The vector describing the incident beam is $\{1, 1, 0, 0\}$, the matrix describing the retarder is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

and the product is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

On referring to Chapter 2, one sees that this last vector, which describes the emerging beam, represents a unit-intensity beam of vertically polarized light.

When, say, four optical devices are inserted in the beam, four

matrices must be used and four multiplications are called for. If the four matrices are called $[M_1]$, $[M_2]$, $[M_3]$, and $[M_4]$, and if $[V_i]$ represents the Stokes vector of the incident beam, the procedure for finding the specification $[V_e]$ of the emerging beam is indicated schematically thus:

$$[M_4][M_3][M_2][M_1][V_i] = [V_e].$$

Consider the experiment in which a unit-intensity beam of left-circularly polarized light is incident on a train containing the following four devices: (1) a linear polarizer with transmission axis horizontal, (2) a linear 90° retarder with fast axis at 45°, (3) a linear polarizer with axis at 45°, (4) a 90° right-circular retarder. The Stokes vector of the incident beam is found from Table 2.1, and the Mueller matrices of the four devices (assumed to be of simple, homogeneous, ideal type) are found from Appendix 2. The multiplication called for is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

On factoring out the fraction $\frac{1}{2}$ from two of the matrices, one obtains

$$\frac{1}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}.$$

One recalls from the standard rules of matrix algebra that the sequence in which the multiplications are performed is important; one must work from right to left. Thus one may begin by multiplying $[V_i]$ by the rightmost matrix $[M_1]$, then multiplying the outcome by $[M_2]$, and so on. On carrying out the four multiplications, one finds the over-all product to be

$$\frac{1}{4}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

This shows that the emerging beam is 100-percent linearly, horizontally polarized and has an intensity of $\frac{1}{4}$.

Obviously, the four devices considered in this example are so simple that a person familiar with polarizers and retarders could predict the

result without recourse to algebra. This would no longer be true, however, if the polarizers were of nonideal type, or if the polarizers or retarders were mounted at unusual angles, or if some of the polarizers or retarders were of elliptical type. The outcome would then be far too complicated to compute in one's head. The Mueller-calculus method, however, *goes through exactly as in the simple case.* The numbers within the matrix are more complicated, but the procedure is identical. Only straightforward arithmetic is used. No special insight or planning is required, other than looking up the matrices, writing them down in proper sequence, and multiplying them starting from the right.

8.4. Mueller Matrix of a Train. It is often advantageous to combine a given series of matrices (representing a given train of optical devices) into one single matrix, called the matrix of the train. One advantage is that the combined matrix gives direct insight into the essential function of the train. A more important advantage is that, once the matrix of the train has been computed, the outcome for *any* given incident beam can be found merely by one multiplication.

Consider the four-member train of the previous section. One temporarily ignores the vector $[V_i]$ and multiplies together the four matrices of the four members of the train. In the present example the product is

$$[M_t] = \tfrac{1}{4}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

This is obviously identical to the matrix of a linear polarizer that has a horizontal transmission axis and a transmittance half that of an ideal polarizer. In other words, the entire train is equivalent to one polarizer. This is true irrespective of the degree and type of polarization of the incident beam. Hence in *all* experiments involving this train (mounted at the given azimuth, and with the given entrance face) the simple expression $[M_t]$ suffices.

Compressing the information into one matrix is, of course, an important simplification in problems that involve a large number of devices and many different choices of incident beam.

As another example of such compression, consider the commercially produced circular polarizer. This consists of a linear polarizer and a

90° linear retarder cemented to it at an azimuth of 45°. If the transmission axis of the polarizer is horizontal and the fast axis of the retarder is at +45° (and if both devices are of ideal type), the matrix of the train is computed thus:

$$[M_t] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

This last matrix thus represents the two-member train. (By referring to Appendix 2, one may see that this matrix *differs* from the matrix of a *homogeneous* circular polarizer.) If the positions of source and observer are interchanged, so that the light is incident first on the retarder and so that the fast axis appears at −45°, the following matrix of the train is obtained:

$$\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

As a final example of compression, consider the series combination of two linear, 180° retarders; assume that the first has a fast axis at 0° and the second at 45°. Then the matrix of the train is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The product is identical to the matrix of a homogeneous, 180°, *circular*, right-handed retarder. Here, then, is a situation where two linear retarders perform exactly as a circular retarder performs.

8.5. Method of Determining the Individual Mueller Matrices. The matrices of the Mueller calculus are not derived from the electromagnetic theory of light or any other theory. Rather, they rest on a phenomenological foundation. They were initially discovered by experimentation, or by reliance on previous experimentation.

Consider, for example, the result of interposing a 180° circular retarder in a beam of linearly polarized light. From experiment one knows that the vibration direction of the light is rotated 90°. This

means that the signs of the second and third elements of the Stokes vector have been changed. Thus, for example, an incident beam $\{1, 0.7, 0.7, 0\}$ results in an emerging beam $\{1, -0.7, -0.7, 0\}$. What 4×4 matrix will convert the former vector into the latter? Persons familiar with matrix algebra will see at once that the following simple matrix accomplishes the purpose:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and this is indeed (see Appendix 2) the Mueller matrix of an ideal 180° circular retarder (of either handedness).

The same approach is used for any other type of retarder, or for any polarizer or other device. One makes a guess as to what the elements of the 4×4 matrix must be, then tries out the matrix to see whether it performs satisfactorily. To be deemed fully satisfactory, it must lead to results that are known to be correct for four independent types of incident beam, for example, (1) unpolarized light, (2) light that is horizontally linearly polarized, (3) light that is linearly polarized at 45°, and (4) light that is right-circularly polarized.

The task of finding the matrix $[P_\theta]$ of a polarizer at some general azimuth θ is easy, once one knows the matrix $[P_0]$ of the polarizer in its principal orientation, that is, with its transmission axis horizontal. Using a conventional procedure of matrix algebra, one computes the product

$$[P_\theta] = [T(-2\theta)][P_0][T(2\theta)],$$

where $[T(2\theta)]$ is the well-known rotator matrix,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & S_2 & 0 \\ 0 & -S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

in which C_2 and S_2 are abbreviations for $\cos 2\theta$ and $\sin 2\theta$, and $[T(-2\theta)]$ is the counterrotator matrix, the same as $[T(2\theta)]$ except that the signs of the sine elements, S_2, are changed.

Performing the indicated multiplication one finds, for the matrix $[P_\theta]$ of an ideal homogeneous linear polarizer with transmission axis at angle θ:

$$[P_\theta] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & -S_2 & 0 \\ 0 & S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2 & S_2 & 0 \\ 0 & -S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & C_2 & S_2 & 0 \\ C_2 & C_2^2 & C_2S_2 & 0 \\ S_2 & C_2S_2 & S_2^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The same procedure applies to a linear retarder, and to an elliptical polarizer and an elliptical retarder: the matrix of a device at a nonprincipal azimuth is computed from that of the device at the principal azimuth with the aid of the rotator matrix and counterrotator matrix.

There is an interesting converse proposition: the matrix of any device at a nonprincipal azimuth can be factored into three matrices, one of which (the central one) describes the device in its principal orientation, while the other two deal solely with rotation.

The importance of rotator matrices is not surprising to persons who are familiar with the Poincaré sphere and remember that any alteration of polarization form is equivalent to *rotating* the sphere suitably.

Appendix 2 lists the more important matrices of the Mueller calculus. Matrices of certain types of scattering objects have been presented by Chandrasekhar and Elbert (C-10), Lenoble (L-18), and van de Hulst (V-1).

8.6. Development of the Mueller Calculus. Professor Hans Mueller formulated his calculus in the early 1940's, at Massachusetts Institute of Technology, and improved it and extended it in the subsequent years. Despite its importance, and its nearly twenty years of life, it has received little mention in published books on optics. However, it has been available in part through references M-26, M-27, and M-28.

The Mueller calculus owes a large debt to several previous men, including G. G. Stokes, P. Soleillet, and F. Perrin. In 1852 Stokes (S-29) invented the vector that bears his name, and found that this vector is a necessary and sufficient description of a beam: any two beams that invariably behave similarly have the same vector, and any two beams that have different vectors will, in certain situations, behave differently.

In 1929 Soleillet (S-24) pointed out that the Stokes vectors trans-

form linearly. To find the change produced in the vector by the interposition of a polarizer, say, one finds each of the parameters S_i of the emerging beam by adding four simple products, thus:

$$S_i = a_1S_1 + a_2S_2 + a_3S_3 + a_4S_4,$$

where S_1, S_2, S_3, and S_4 are the parameters of the incident beam and a_1, a_2, a_3, and a_4 are constants of the given polarizer in its given orientation.

Perrin (P-12) showed that the linear relations could be put into matrix form. Mueller carried forward the joining of the Stokes vectors and the 4×4 matrix forms, worked out a large number of matrices, placed the calculus on a firm phenomenological basis, and demonstrated the power of the calculus in solving various problems that had previously appeared virtually insoluble.

In 1948 Parke (P-6, P-7, P-8, P-9) explored the applicability of the new calculus to unpolarized light, and succeeded in relating it rigorously to the electromagnetic theory. Van de Hulst (H-1) applied the calculus to various complicated types of light scattering. Chandrasekhar and Elbert (C-260) and Lenoble (L-18) also gave attention to this subject. Fano (F-5) and McMaster (M-2, M-3, M-5) applied the 4×4 matrices to the scattering of gamma radiation. Other pertinent articles have been published by Jones (J-25, J-33), Billings (B-24), Billings and Land (B-22), Walker (W-3), and Weeks (W-11).

8.7. Outline of the Jones Calculus. The Jones calculus, invented in 1940 and 1941 by R. Clark Jones (J-19, H-40, J-20, J-21, J-25, J-26, J-27, J-33), is another treatment in which the incident light is described by a vector, the optical device is described by a matrix, and the outcome is computed by multiplying the vector by the matrix. However, the Jones calculus has the advantage over the Mueller calculus of employing a smaller matrix (2×2, instead of 4×4), and is applicable even to problems in which information as to phase must be preserved. On the other hand, many of its matrix elements are complex. Also, it is entirely inapplicable to optical devices that have depolarizing tendencies. Thus in several respects the Jones calculus complements, rather than competes with, the Mueller calculus. (The relation has been explored at length by Parke, P-6, P-7, P-8, P-9.) Section 8.9 presents a detailed review of the strengths and weaknesses of the two kinds of calculi.

In using the Jones calculus, one specifies the incident beam in terms of its Jones vector, specifies the various polarizers and retarders encountered by means of the appropriate Jones matrices, then multiplies these expressions to obtain the Jones vector of the emerging beam.

The Jones vector has been explained in Sec. 2.4, and the most important examples are indicated in Table 2.1. For example, horizontally polarized light and right-circularly polarized light have the *full Jones vectors*

$$\begin{bmatrix} A_x e^{i\epsilon_x} \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} A_x e^{i\epsilon_x} \\ A_x e^{i(\epsilon_x + \frac{1}{2}\pi)} \end{bmatrix}$$

and have the simplified ("standard normalized") forms

$$\begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} -\dfrac{i}{\sqrt{2}} \\ \\ \dfrac{1}{\sqrt{2}} \end{bmatrix}.$$

The Jones matrices of the most important polarizers and retarders are listed in Appendix 2. Each matrix describes a given device in a given orientation, and assumes that a given face serves as entrance face. The matrices are derived from the usual mathematical expression for a monochromatic (polarized) wave train and from mathematical analysis of the changes produced by interposing a given polarizer or retarder; the derivations are explained in Refs. J-19 and J-26.

The matrices presented in Appendix 2 are in the simplest form and are called standard matrices. They are ideally designed for use by an investigator interested in the intensity and polarization form of an emerging beam. However, they contain no information as to the change produced in the absolute phase. An investigator wishing to determine the absolute phase of an emerging beam must use the *full matrix*. For example, the full matrix of an ideal homogeneous linear polarizer oriented with its transmission axis horizontal is

$$\begin{bmatrix} e^{-i2\pi nd/\lambda_v} & 0 \\ 0 & 0 \end{bmatrix} \quad \text{or} \quad e^{-i2\pi nd/\lambda_v} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

where n is the pertinent refractive index, d is the thickness, and λ_v is the wavelength in vacuum. Obviously, the term $e^{-i2\pi nd/\lambda_v}$ carries the information as to absolute phase.

The matrix $[P_\theta]$ of a polarizer at some nonprincipal azimuth θ is

easily obtained from the matrix $[P_0]$ of the polarizer in its principal azimuth. As in the case of the Mueller calculus, the procedure involves a rotator matrix and a counterrotator matrix. These latter, called $[S(\theta)]$ and $[S(-\theta)]$, have the forms

$$[S(\theta)] = \begin{bmatrix} C_1 & S_1 \\ -S_1 & C_1 \end{bmatrix} \quad \text{and} \quad [S(-\theta)] = \begin{bmatrix} C_1 & -S_1 \\ S_1 & C_1 \end{bmatrix},$$

where C_1 and S_1 are abbreviations for $\cos\theta$ and $\sin\theta$ respectively. The relation between $[P_\theta]$ and $[P_0]$ is

$$[P_\theta] = [S(-\theta)][P_0][S(\theta)].$$

A corresponding relation applies to the nonprincipal matrix $[R_\theta]$ of a retarder and the principal matrix $[R_0]$ of the retarder. Again, the $[R_0]$ matrix describes the retarder in its principal orientation, and hence may be regarded as the simplest, most basic description of the retarder.

To find the result of interposing several optical devices in a beam of 100-percent polarized light, one writes down (at the right!) the Jones vector of the incident beam, and then writes down the Jones matrices of the various devices. Again the sequence of the matrices is important, and must correspond to that of the matrices themselves; the matrix of the last-to-be-encountered device must appear at the left.

8.8. Examples of Applications of the Jones Calculus. Consider the problem presented in the last part of Sec. 8.3: a unit-intensity horizontally polarized beam strikes an ideal homogeneous linear 180° retarder whose fast axis is at 45°; what is the character of the emerging beam? Since the Jones vector of the incident beam is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and the Jones matrix of the retarder is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, the answer is found by obtaining the product:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

On referring to Table 2.1, one finds that the resulting expression is the Jones vector of a beam of vertically polarized light.

To solve the problem presented in Sec. 8.4 is simple also. One looks up (in Table 2.1) the Jones vector of the incident beam, which is assumed to be a unit-intensity beam that is left-circularly polarized.

One then looks up (in Appendix 2) the Jones matrices of the four ideal homogeneous optical devices in question, namely, (1) a linear polarizer with transmission axis horizontal, (2) a linear 90° retarder with fast axis at 45°, (3) a linear polarizer with transmission axis at 45°, and (4) a 90° right-circular retarder. Thus one finds that the multiplication called for is

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}\,i}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

or

$$\frac{\sqrt{2}}{8} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ 1 \end{bmatrix}.$$

When the multiplication has been performed (using the proper sequence, as discussed in Sec. 8.4), the answer arrived at is

$$\frac{\sqrt{2}}{8} \begin{bmatrix} 2i - 2 \\ 0 \end{bmatrix} \quad \text{or} \quad \frac{\sqrt{2}}{4} \begin{bmatrix} i - 1 \\ 0 \end{bmatrix} \quad \text{or} \quad \frac{1}{2} e^{-i\frac{3\pi}{4}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Reference to Sec. 2.4 shows that this answer corresponds to a horizontally polarized beam whose intensity is $(\frac{1}{2})^2$ or $\frac{1}{4}$.

As was the case for the Mueller calculus, so here too the investigator may find it desirable to compute the over-all *matrix of the train*, then multiply the incident beam's vector by this. When a large number of alternative forms of incident beam are to be considered, this procedure saves much time.

N-Matrices. The ordinary matrix of the Jones calculus is concerned with an optical device *as a whole*, and gives no information as to either the polarization form at an *intermediate* location within the device or the *change* of polarization form that is produced per unit pathlength within the device. However, information of this type is provided by what Jones (J-27) has called an N-matrix, a matrix that is obtained from the appropriate standard matrix (M-matrix) by a process involving differentiation. Obviously, the N-matrix describes the intensive properties of the device, and is indeed a most succinct and informative specification of these properties. Each of eight major types of optical behavior of crystals has its own type of N-matrix. By examining the N-matrix of a given crystal, one can determine the average values of

the refractive index and the absorption coefficient, the circular birefringence and circular dichroism, and the linear birefringence and linear dichroism with respect to the coordinate axes and with respect to the bisectors of those axes. The method has been explained by Jones (J-27) and by Ramachandran and Ramaseshan (R-2).

Hsu, Richartz, and Liang (H-34), Evans (E-17), and Dawson and Young (D-3) have applied the Jones calculus to trains of linear retarders.

8.9. Comparison of Mueller Calculus and Jones Calculus. The two kinds of calculi have much in common. Each describes the light beam in a standard manner. Each leads to an answer by a standard kind of process, namely, an elementary process of matrix algebra. The investigator follows a fixed routine in which little thought is required beyond looking up the vectors and matrices in a table and performing the standard multiplication operations.

Each kind of calculus permits the investigator to find a single *matrix of the train* and thereby obtain the most succinct statement of the essential function of the train. Also, each calculus employs rotator matrices.

There are, however, important differences:

1. The Mueller calculus can handle problems involving depolarization. The Jones calculus cannot.
2. The Mueller calculus is based on a phenomenological foundation, hence does not depend on the validity of the electromagnetic theory. The Jones calculus is derived directly from that theory.
3. The Jones calculus permits one to preserve information as to absolute phase. The Mueller calculus does not; indeed it categorically pays no heed to phase.
4. The Jones calculus is well suited to the handling of problems involving the combining of two beams that are coherent. The Mueller calculus is unable to do so, except perhaps with great difficulty.
5. The Mueller calculus employs a vector (Stokes vector) whose first term indicates the intensity directly. The vector employed by the Jones calculus does not do so; to find the intensity one must obtain the sum of the squares of the elements.
6. The Jones matrices employ elements associated with amplitude transmittance. The Mueller matrix elements are associated with intensity transmittance.

7. The Jones calculus is well suited to problems involving a large number of similar devices that are arranged in series in a regular manner, and permits an investigator to arrive at an answer expressed explicitly in terms of n, the number of such devices; see Chapter 9. The Mueller calculus is not suited to such purpose.

8. The Jones matrix of a train of absorbing or nonabsorbing, nondepolarizing polarizers and retarders contains no redundant information: the matrix contains four elements comprising eight constants, and none of these constants is a function of any other. The Mueller matrix of such a train contains much redundancy: sixteen constants appear, and only seven of these are independent (J-25).

9. The Jones matrix of a birefringent dichroic device may be differentiated to reveal information as to the device's intensive properties. The Mueller calculus — in practice — lacks this capability.

Parke has analyzed and compared the two calculi in detail (P-6, P-7, P-8, P-9).

8.10. Related Topics. Matrix methods have been considered also by Fano (F-5) and by McMaster (M-2, M-3, M-5), and have been applied by them to the polarization of gamma rays by means of Compton scattering. In 1959 Westfold (W-23) considered an alternative method of describing polarized light in terms of complex vectors, and applied the method to the Faraday effect exhibited by radio waves. In 1949 Richartz and Hsu (R-7) described a calculus relying on quaternions instead of matrices. Weeks (W-11) derived a series of 4×4 "coherency matrices" that assist the solving of certain types of problems.

9 APPLICATIONS TO THE CONTROL OF INTENSITY, GLARE, AND COLOR

9.1. Introduction. Here and in the following chapter the main applications of polarizers and polarized light to science and technology are considered. (Brief lists of applications may be found in various papers published ten or twenty years ago, including Refs. L-8, G-11, G-12, M-13, P-22, S-12, W-22.) The applications are arranged according to the kind of function performed by the polarizer. The simplest function — that of producing a general, indiscriminate reduction in light intensity — is considered in the Secs. 9.1–9.3. Later sections deal with the suppression of polarized glare, the forestalling of the initiation of such glare, the enhancement of contrast, various polarization-coding schemes used in presenting stereoscopic motion pictures and stereoscopic television, and the control of spectral energy distribution and color.

Chapter 10 deals with applications in which polarization is inherently germane, and thus necessarily plays a central role.

Many references are cited. By referring to them, a reader may gain quick access to detailed information on any given topic.

9.2. Control of General Intensity. As a means of controlling the general intensity of a beam of light, a pair of linear polarizers has many advantages over the principal alternative means, such as the variable diaphragm, the step wedge, and the graduated comb. The pair of polarizers makes it possible to vary the attenuation smoothly and throughout an enormous range, as great as 100,000 to 1. Furthermore,

the attenuation obeys a fixed and well-known law, namely, the law of Malus:

$$I = I_0 H_0 \cos^2 \theta,$$

where I_0 is the intensity before the pair of polarizers is inserted, H_0 is the transmittance of the parallel pair (see Sec. 3.5), and θ is the angle of crossing. (If the quantity H_{90} differs appreciably from zero, a slightly more complicated formula applies; see Sec. 9.10.) The pair of polarizers treats the entire cross section of the beam uniformly, so that no change in linear or angular aperture occurs. In some applications, notably the Kerr shutter, the attenuation of the pair of polarizers is governed by electrically controlled changes in the individual molecules of an intervening retarder.

Perhaps the simplest application of the pair of linear polarizers is to variable-density sunglasses, developed by the Polaroid Corporation prior to World War II. As explained by Grabau (G-12), each window of the sunglasses contains a pair of polarizers: the outer one is fixed, with its transmission axis vertical; the other can be turned to any desired azimuth by means of a small control lever. Thus any attenuation can be achieved. The two pairs (for the two eyes) are linked together so that the changes produced are the same for each eye. Specific designs of variable-density sunglasses have been proposed by Land (U.S. patent 2,005,426) and by Archambault (2,813,459).

Railroad-train windows consisting of pairs of polarizers 30 in. in diameter were introduced in 1938 (G-12). By turning the inner polarizer relative to the fixed outer polarizer, the passenger can achieve almost any desired attenuation in the light, without reducing the effective width or height of the window. In 1953 a number of pair-of-polarizer windows were installed in the ocean liners *Independence* and *Constitution.*

Various kinds of spectrophotometers employ pairs of polarizers as photometering element, that is, the element that reduces the intensity of the reference beam and increases the intensity of the sample beam in order that the absorption by the sample may be exactly compensated. Since the attenuation depends in a known manner on the azimuth of the rotatable polarizer, the spectrophotometer designer can easily design a cam system that will convert values of polarizer azimuth into values of sample transmittance. The Martens photometer employs a pair of polarizers, and the same is true of the General Electric Company spectrophotometer.

If a photometer designer plans to use dichroic polarizers, and if the family of polarizers in question has a given value of dichroic ratio ($R_d = d_2/d_1$), the designer is still free to specify the d_2 value the polarizers shall have. It is easily shown that an intermediate value leads to the best over-all performance of the photometer. Formulas for computing the optimum value of d_2 have been derived by Jones and West (J-29).

British patent 796,661 of 1958 by the Carl Zeiss Company describes a spectrophotometer in which special dual-purpose prism-type polarizers are used. Besides producing polarization, they perform the functions of beam splitting and beam combining without introducing dispersion troubles such as arise when a Wollaston-prism beam splitter is used.

A photometer employing a train of three polarizers has been described by Howard, Hood, and Ballard (H-33). The train obeys a cosine-fourth law, and covers a density range of 8. Because the device uses white light, rather than monochromatic light, it avoids the *Jones anomaly* (J-32) that may arise when three polarizers are used and the intermediate one exhibits some birefringence.

Hulburt (H-37) has described the application of pairs of polarizers to sextants; by adjusting the polarizers, the user can dim the sun's disk to any desired extent. Operators of cathode-ray oscilloscopes sometimes use pairs of polarizers in front of the screen in order to change the brightness without altering any of the electrical controls (S-14); changing those controls may affect the focus of the electron beam. Photographers employ polarizers in studio luminaires and enlarging cameras, to vary the light intensity without altering the lamp voltage and hence without altering the spectral energy distribution of the emitted light.

The Kerr-cell shutter, invented in 1875 by the Scottish physicist John Kerr (K-9), employs a pair of polarizers and a variable retarder. As indicated in Fig. 9.1, the polarizers are fixed, with transmission axes at 0° and 90°. The variable retarder is situated between them. When not subjected to an electric field, the retarder has no birefringence and hence no retardance; but when subjected to an intense electric field, the material becomes linearly birefringent and the retardance may amount to 180° or more. The applied field is at 45°; accordingly, when a retardance of 180° is achieved, the vibration direction of the light emerging from the retarder is changed by 90° from the

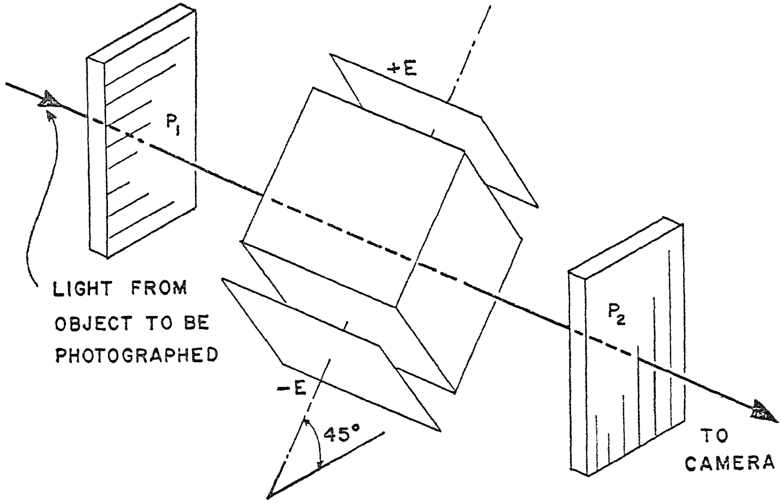


FIG. 9.1. Diagram of Kerr shutter. The polarizers P_1 and P_2 have transmission axes at 0° and 90°. The electric field across the nitrobenzene-filled box is at 45°. In practice, polarizers and box form a short, compact assembly.

vibration direction of the light incident on the retarder. Thus the transmittance of the entire system is simply H_0, the transmittance of the parallel pair of polarizers. When the field ceases, the retardance reverts to zero, and the transmittance of the system drops to zero (more exactly, it drops to H_{90}, the transmittance of the *crossed* pair).

The retarder consists of a transparent body composed of a material having a large Kerr constant (P-10, A-7, L-17). Usually mononitrobenzene, a liquid, is used; it is kept in a rectangular glass box situated between the two polarizers (Fig. 9.1). The relation between retardance δ and field strength E is

$$\delta = 2\pi LKE^2,$$

where L is the pathlength of the light beam in the liquid and K is the Kerr constant of the liquid.

The transmittance T of the shutter is related to the retardance, and hence to the applied potential V, thus:

$$T = H_0 \sin^2 \tfrac{1}{2}\delta_{\text{deg}} = H_0 \sin^2 [(V/V_o)^2 90°].$$

Here V_o is that potential which produces a retardance of 180°. Because T depends on the *square* of V, rather than the first power, the time constant of the transmittance is shorter than that of the applied field, as explained by Nicholson and Ross (N-4). When potential dif-

ferences as great as 45 kv are employed, the retardance may be altered very rapidly, even in an interval as short as 10^{-7} or 10^{-8} sec (N-4, S-32, T-3, Z-3).

In the foregoing equation, the assumption is made that the H_{90} value of the polarizer is zero. In practice this quantity is of the order of 10^{-5} or 10^{-6}, with the result that, over a period of many hours, enough energy may pass through the shutter to fog the photographic film. To avoid this danger one employs an auxiliary mechanical shutter that is opened only when the Kerr shutter is about to be energized.

Instead of employing an electric field that is transverse to the beam, one may employ a field that is parallel to the beam. As retarder, an acentric cubic crystal having large Pockel's effect is used, for example, cuprous chloride, discussed by West in U.S. patent 2,788,710. (Ammonium dihydrogen phosphate may be used successfully if the beam has small angular aperture; see Ref. C-3.) Reference may be made also to high-speed shutters proposed by Baerwald (patent 2,766,659) and by Bond (patent 2,768,557). In proposing new designs, the designer's usual goal is to increase the permissible angular width of the beam or to decrease the time constant.

Magneto-optical shutters have been developed also. These too employ crossed polarizers between which is a variable retarder, consisting, ordinarily, of a thick plate of flint glass. The retardance is controlled by means of a magnetic field provided by a solenoid; the field is parallel to the beam, and thus takes advantage of the Faraday effect. The theory of such shutters has been explained in detail by Edgerton and Wyckoff (E-5). In one of their designs a train of three polarizers is used, in order that the over-all transmittance of the train will be extremely low (less than 10^{-8}) when the shutter is unenergized.

In some magneto-optical shutters the magnetic field is perpendicular to the beam, and thus operates by virtue of the Cotton-Mouton effect; however, this effect is very small and shutters of this type are seldom used.

An unusual design of fast-acting shutter is that of Porter, Spencer, and LeCraw (P-29). Using a variable retarder consisting of a 0.004-in. section of yttrium iron garnet, they succeeded in modulating the light beam efficiently at 60 kc/sec, and they concluded that a modified design could operate successfully at microwave frequencies. Another unusual design is that due to Billings (B-21); here the retardance is varied by varying the mechanical stress exerted on the retarder.

9.3. Light-Lock Illumination System. A light-lock illumination system is an illumination-control system that governs the passage of light among three regions A, B, and C, in such a manner that light can pass readily from A to B and vice versa and can pass from B to C and vice versa, but cannot pass from A to C or from C to A. Consider, for example, a motion-picture theater: the auditorium, lobby, and adjacent sidewalk area may be called regions A, B, and C. It is permissible and even desirable that there by visibility across the boundary from A to B and across the boundary from B to C; yet any passage of light from C to A may interfere with the enjoyment of the movie. Satisfactory control may be accomplished by using transparent panels or doors made of linear polarizers: panels with transmission axes horizontal are installed between A and B, and panels with vertical axes are installed between B and C. Such systems have been discussed by Grabau (G-12). They derive their name from the similarity to the lock in a canal: here too the purpose is to join A and B and join B and C, while always isolating A from C.

A similar scheme has been used to advantage in combat-information centers. Such centers usually include radar screens, and the radar operator B must see screen A and also various maps and the like illuminated by an overhead lamp C; yet passage of light from C to A will reduce the contrast of the screen images. White (W-24, also patent 2,793,361) and Henry (H-22) have shown that the problem can be solved satisfactorily by equipping the lamp and the screen with polarizers having orthogonal orientations. Kraft (K-17) has pointed out that color-coded filters are superior to polarization-coded filters in certain situations; however, color-coded filters are scarcely applicable in situations where the screen-image is multicolored or where the operator must work with multicolored maps and charts (see W-24).

The light-lock illumination system could be applied to dwellings that face one another across a courtyard. If the opposing windows are equipped with orthogonal polarizers, occupants of each dwelling can see into the courtyard but not into the opposite building.

9.4. Control of Headlight Glare. Automobile drivers would find night driving easier on the eyes, and also safer, if the light-lock illumination principle were applied to automobiles. As early as 1920, F. Short (patent 1,734,022) and also L. W. Chubb (patent 2,087,795, applied

for in 1920) pointed out the potentialities of the polarization-type light-lock system; when Land invented the sheet-type dichroic polarizer, the realization of the earlier proposals seemed much closer at hand. With the invention of K-sheet in about 1939, the last major technical obstacle was removed.

To apply the light-lock illumination principle to automobiles, one would equip each car with two sets of polarizers. In one easily described (but far from optimum) scheme, one would install polarizers having horizontal transmission axes in front of the headlights and install polarizers having vertical transmission axes in front of the windshield or in a small "visor" situated just in front of the driver's eyes. When one car is approaching another along a level road, the road is illuminated by the headlights of both cars. A majority of the objects illuminated depolarize the light to a considerable extent, and accordingly each driver can see these objects adequately by the light from either car or, of course, from both. However, neither driver receives light directly from the other car's headlights. Even if both cars employ especially powerful headlights, and even if these are operated always on "high beam," no direct glare from the approaching car's headlights is experienced. (The polarizers used in driving tests arranged by the Polaroid Corporation have a small but appreciable value of crossed transmittance H_{90}, a value just great enough that the approaching car's headlights are adequately noticeable.)

Many modifications of this basic approach have been proposed by Land (L-4, L-5, L-7, L-11, L-12), Billings and Land (B-22), and others. The modifications have the goals of maximizing the driver's utilization of light from his own headlights and maximizing the contrast of a typically clothed pedestrian relative to a typical roadway. Another goal is to minimize the (tilted) windshield's tendency to (*a*) produce double imaging as a result of multiple reflections within the thick glass plate, and (*b*) alter the polarization form systematically by virtue of the *oblique* tilt of the windshield area directly in front of the driver's head.

A possible complication is strain in the windshield glass. If the strain is very large, the windshield will act as a retarder; consequently it will alter the polarization form of the light transmitted and defeat the light lock. Placing the visor in front of the windshield would solve this problem but create others. A more straightforward solution is to make sure that the windshield is free of any large strains.

One of the superior schemes calls for orienting all the polarizers at 45°, so that the axes of a given car's headlight polarizers and visor polarizers are parallel; if the windshield were perpendicular to the car's line of travel, this scheme would maximize the driver's utilization of light from his own headlights, assuming that the objects illuminated were of polarization-conserving type. Another scheme, called the "minus 55°, minus 35° " system, employs headlight polarizers whose axes (as judged by the driver) have an upward-to-the-left, downward-to-the-right direction at 55° from the vertical; the axis of the visor also has an upward-to-the-left, downward-to-the-right direction, although here the direction is at 35° from the vertical; this scheme minimizes various complications associated with windshield obliquity.

Billings and Land (B-22, P-24) have shown that, in fact, there are an infinite number of satisfactory systems. To a first approximation, any linear-polarizer scheme will be successful if the axes of the visor polarizer and the headlight polarizer are symmetric with respect to a line making an angle of 45° with the vertical.

Some consideration has been given to schemes employing circular polarizers (see B-22 and also Land's patent 2,099,694). Good performance would result if headlights and visors were equipped with, say, right- and left-circular polarizers respectively. An advantage of this system is that the extent of glare suppression is invariant with respect to tipping of the car, as on a highly crowned road. There are practical disadvantages, however; circular polarizers are difficult to make with the required extent of uniformity; also, they often have large chromatic effects (variation of retardance δ_{deg} with wavelength) and may have significant obliquity effects.

Linear polarizers have been favored in most of the studies made, such as those by the Polaroid Corporation (P-24, B-22) and Roper and Scott (R-11). K-type polarizers are preferred for headlights, as they are well suited to standing the high temperatures involved there. Either K or H polarizers may be used for the visor. If the individual polarizers have total luminous transmittance values of approximately 38 to 40 percent, the headlight power must be increased by a factor of about 3 if the headlight aim remains unchanged and the same amount of light is to reach the driver's eye; however, when the polarizing system is in use, the headlight aim can ordinarily be left at "high beam," so that — even with no increase in power — the illumination of ob-

jects at a considerable distance will exceed that which prevails when no polarizers are used and "low beam" is employed.

The benefit that polarizing systems can provide is great, but exactly how great would be difficult to predict. If, for example, the most important goal is deemed to be that of detecting a pedestrian several hundred feet ahead of the car, the benefit can be estimated only after information has been obtained as to the reflectance of typical roads and typical clothing and the extents to which road and clothing conserve the polarization form. Such factors have been considered by Roper and Scott (R-11), Laurence (L-15), and Billings and Land (B-22); the results indicate that the benefit would indeed be great.

To inaugurate country-wide use of a polarizing headlight system would entail educating the public to its advantages, arranging the passage of suitable laws and regulations, and enlarging the facilities for producing K-sheet. To provide a smooth transition from the present (nonpolarizing) system to the proposed polarizing system would require careful planning and perhaps special interim equipment. Specific plans have been proposed by Land (L-9, L-10) and Roper and Scott (R-11).

Why has no polarizing system been adopted? Perhaps apathy on the part of the public and the automobile manufacturers is the main cause. The extra expense of polarizers, visors, and so forth, is another reason, but probably a minor one in view of the low cost of polarizers relative to other common accessories on automobiles. In a 1952 pamphlet (A-22) the Automobile Manufacturers Association listed certain technical objections, including (*a*) the possible need to furnish visors for the passengers, as well as for the driver, (*b*) residual glare from light reflected off the sides of intervening cars, (*c*) glare that may be experienced by pedestrians.

Some commercial organizations have urged the use of tinted (isotropic) glasses and tinted windshields to reduce the glare. However, Blackwell (B-30) and Haber (H-5) have shown that such devices may do more harm than good.

Polarizing systems have been considered in other countries also. In 1956 Jehu (J-7), in Germany, proposed that polarizers be used for low-beam headlights but not for high-beam headlights; the driver would use the former when a car is approaching, and the latter when no car is approaching. The work of Weigel (W-12) should be mentioned also. In England, the proposal has been made (according to Jehu)

that each headlight beam be of mixed type; the rays having high aim would be polarized, while those having low aim would be left unpolarized.

Many inventors have proposed using specially designed pile-of-plates polarizers in headlight systems, to avoid the absorption that is inevitable in dichroic polarizers. However, difficulties of bulk, fragility, tendency to become cloudy, polarization defect, and manufacturing cost necessarily arise.

Nighttime Driving on Fog-Shrouded Highways. Under certain conditions of fog, nighttime driving can be assisted greatly by use of polarizers, according to unpublished reports by D. S. Grey and L. W. Chubb of the Polaroid Corporation and recent tests made by Nathan (N-1). The benefit becomes small, however, if the fog is dense, or if the ambient illumination is appreciable. In 1957 Pritchard and Blackwell (P-32) investigated the effectiveness of polarizers mounted in street lamps as a means of improving nighttime driving in fog. They found that, under a variety of fog conditions, much improvement was made if the polarizer-equipped street lamps were aimed almost straight downward and the drivers were provided with polarizing visors.

9.5. Eliminating Light That Has Been Specularly Reflected from a Smooth, Oblique, Dielectric Surface. When unpolarized light strikes the surface of a pond obliquely, a portion of the light is, of course, specularly reflected. Such reflected light makes it difficult for a fisherman, say, to see fish, rocks, and so forth, situated beneath the surface. However, if the fisherman views the pond through a suitably oriented linear polarizer, a large fraction of the specularly reflected light is absorbed by the polarizer, and accordingly the visibility of the underwater objects is greatly improved. The polarizer is effective because the specularly reflected light is linearly polarized to a considerable extent; when the light is incident at Brewster's angle (Sec. 6.2), the degree of polarization is 100 percent. Since the surface of the pond is horizontal, the dominant vibration direction of the reflected light is horizontal, and the polarizer's *absorption* axis should be horizontal.

Polarizing sunglasses are designed accordingly. Each "lens" is a linear polarizer whose absorption axis is horizontal. As polarizer, HN-32 is often used; an isotropic green dye may be added, to produce a color suggestive of coolness and to eliminate ultraviolet and near-

infrared radiation. A fisherman who dons such sunglasses and looks obliquely downward toward the surface of the water finds that the "noise" — the surface reflection of light from the sky — is reduced by a factor of perhaps 5 to 20, whereas the signal — the light from the underwater objects — is reduced by a factor of 2 to 4. Consequently the signal-to-noise ratio is greatly improved.

An automobile driver, gazing obliquely downward toward a smooth road, finds a comparable, though perhaps smaller, improvement.

Leaves, blades of grass, and the like tend to partially polarize light that is incident obliquely; the leaves that produce the most intense specular reflection are those that are tilted in such a way as to specularly reflect light from sun and sky toward the observer; thus the dominant vibration direction is again horizontal, and the same design of polarizing sunglasses is effective. When viewed through neutral (gray) polarizing sunglasses, foliage, grass, and flowers appear softer and more highly colored than when viewed without benefit of polarizers.

Polarizing windows have been used in airport control towers, to improve the visibility of airplanes landing and taking off. Polarizers may be used successfully in binoculars, according to Grabau (G-12).

Instead of wearing polarizing sunglasses, one may incorporate a single, large-area polarizer in the visor of a cap; the visor may be swung out of the way when not needed. In automobiles, a polarizing visor may be mounted in place of, or adjacent to, the usual visor, and can be brought into play or swung out of the way at will. Designs of polarizing visors for use in automobiles have been described by Land (patents 2,102,632 and 2,237,565) and Frost (patent 2,856,810).

Photographers make much use of polarizers. In this section we consider only the application to the elimination of specular reflections from objects being photographed, for example, chinaware, drawings, paintings, tables, floors, cellophane-wrapped objects (see E-1, E-4, H-3). By equipping his camera with a suitably oriented polarizer, the photographer can obtain pictures that are nearly free of specular reflections and hence portray the objects' "body color" more faithfully. Fallon (F-4) has discussed the use of polarizers in photographing surgical operations. Polarizers are used also in assembly lines and inspection stations, to improve the visibility of small defects. They may be used in photographing show-window displays to eliminate

troublesome reflections from the intervening plate-glass window. They assist the photographing of underwater objects from an airplane.

In each case the polarizer is turned to the azimuth appropriate to blocking the unwanted polarization form; the transmission axis of the polarizer should be vertical if the specularly reflecting surface is horizontal, and vice versa. In general, the photographer may adjust the polarizer by rule of thumb or he may remove the polarizer from the camera, look through it by eye, and visually judge which azimuth is optimum. Some manufacturers market a combination of a camera polarizer and an outrigger-mounted viewing polarizer; the two polarizers are aligned, so that if the outrigger is turned so as to bring the viewing polarizer to optimum azimuth the azimuth of the camera polarizer will be optimum also. Thus the latter need not be removed from the lens assembly for determination of azimuth.

9.6. Forestalling the Specular Reflection of Light. Instead of interposing a polarizer in the beam after the specular reflection has occurred, one may interpose it between light source and specimen in such a way as to prevent any specular reflection. If the light source strikes the smooth dielectric specimen at Brewster's angle, and if the polarizer is turned so that its *absorption* axis is parallel to the surface in question, almost none of the light will be specularly reflected. It is then unnecessary for the observer to employ sunglasses or visor.

The method has been applied successfully to photographic-studio lighting, inspection-station lighting, photography of surgical operations, and the illumination of paintings in art galleries (G-12). It has been applied also to a desk lamp that contains a built-in polarizer (Land's patent 2,302,613). More recently it has been applied to the lighting of offices; for this purpose Marks (M-15) has employed multilayer reflection-type polarizers.

9.7. Enhancing the Contrast of Clouds. Light from clouds exhibits little or no polarization, but light from the clear blue sky is linearly polarized to a considerable degree (see Sec. 10.9). When viewing a sky containing clouds and also clear blue areas, one may enhance the contrast between clouds and blue areas by interposing a linear polarizer oriented at a suitable azimuth. The usual rule (E-2, E-3) is to turn the polarizer so that its transmission axis lies in the plane determined

by sun, observer, and object being photographed. In taking black-and-white pictures of clouds and sky, one finds the polarizing filter to be very effective, especially if the camera is aimed roughly perpendicular to the sun's rays; however, isotropic yellow or orange filters also enhance the contrast. When taking *colored* pictures of sky and clouds, the use of yellow or orange filters is, of course, ruled out; thus the (neutral-colored) polarizing filter is the sole optical means of contrast enhancement.

9.8. Suppressing Perpendicularly Reflected Light. Previous sections have shown how a linear polarizer may be used to eliminate light that has been specularly reflected (from a dielectric surface) at Brewster's angle. The method is inapplicable, however, if the incident light strikes the surface perpendicularly. Fortunately, the circular polarizer solves this problem (S-14). If right-circularly polarized light strikes a sheet of glass or shiny metal perpendicularly, the specularly reflected beam will be found to exhibit *left*-circular polarization (since the direction of propagation has been reversed, changing the right-helical snapshot pattern into a left-helical pattern). Accordingly, the same polarizer that produces the circular polarization of the incident beam will *block* the returning beam. One may regard the polarizer as performing a coding operation on the incident beam; the specularly reflecting surface reverses the coding, and accordingly the polarizer — now serving as analyzer — blocks the returning beam, as shown in Fig. 9.2. (In this figure, for clarity of exposition, the incident and reflected beams are shown as having slight obliquity and following slightly different paths. If they are strictly perpendicular to the surface in question and are coincident, the effectiveness of the circular polarizer is, of course, equally great — indeed, greater.)

When the incident light is truly circularly polarized and is incident exactly along the normal, and when the reflecting surface is a polished plate of glass or metal, the suppression of the round-trip rays is almost complete; the quantity k_{ntrt}, defined in Sec. 3.7, may be less than 10^{-4}. Under less ideal conditions, values of 10^{-1} to 10^{-2} are typical, as indicated in Fig. 7.9.

(If the glass plate is mat, rather than specularly reflecting, the incident light is depolarized to a considerable extent; hence only a fraction of the returning beam is blocked. If the transparent plate is made of highly birefringent material, the polarization form of the

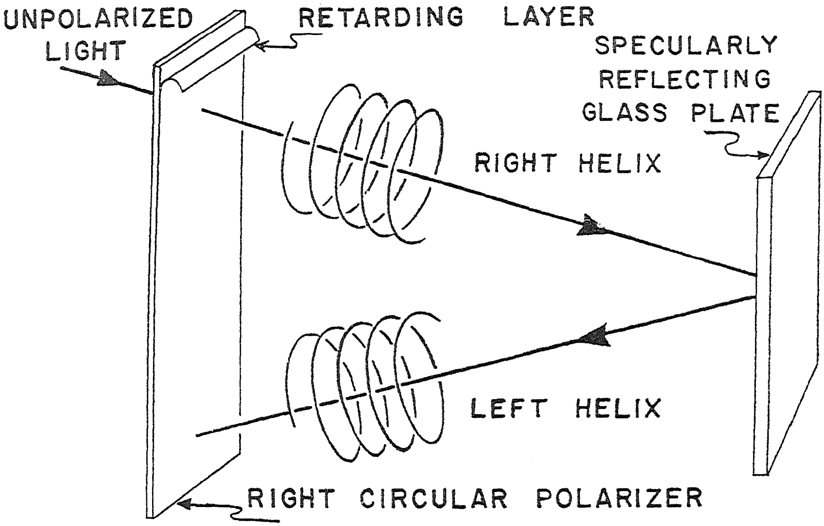


FIG. 9.2. Use of a right circular polarizer in suppressing light that has been specularly reflected approximately perpendicularly from a glass or metal plate.

light that enters the plate and is reflected from the farther surface may be altered; hence the coding is spoiled and much of the returning energy may succeed in passing through the polarizer.)

Circular polarizers have been applied with much success to the suppression of specular reflection from cathode-ray-oscilloscope screens in general and radar screens in particular. The need for a suppression scheme is obvious: the trace appearing on the screen may be faint and the room lighting may be intense; consequently the specular reflection of room light from the outer surface of the screen or from the protecting cover plate may seriously reduce the visibility of the trace (F-10). White (W-24) has made an analysis of the contrast improvement factor achieved using circular polarizers positioned just in front of the radar screen. He found the factor to be large — larger than when color coding is used instead of polarization coding. His patent 2,793,361 describes an illumination-control system that entails equipping the oscilloscope screen with a circular polarizer and equipping the overhead lights with linear polarizers. The circular polarizer is of the usual two-layer type (Sec. 7.10); the outer layer is, of course, the linear layer, and this is oriented so as to be crossed with the linear polarizers associated with the overhead lights. Other investigators,

also, have explored the benefits that result from employing circular polarizers (C-21).

Circular polarizers may be used on television screens. Here, too, specular reflection of room light is suppressed, and the contrast and general visibility of the picture is improved. Circular polarizers may be applied also to traffic lights, to prevent confusing effects associated with reflection of ambient light (Land's patents 2,018,963 and 2,334,418). Reflections from the glass windows on voltmeters, ammeters, and the like, may be suppressed in a similar manner. In every instance the prime face of the circular polarizer — the retarder-layer face — must be *toward* the offending reflecting surface. Of course, troublesome reflections from the polarizer itself must be guarded against, either by applying low-reflection coatings to the polarizer or by tilting it slightly downward toward, say, a black cloth.

The same general principle may be used in microwave radar, to suppress energy that has been reflected from clouds and fogs. The (special) antenna emits circularly polarized waves; energy reflected by the water droplets of a cloud is reversed in handedness, and hence in effect is not collected by the antenna; but energy reflected from typical targets is depolarized to an appreciable extent, and hence is collected by the antenna with reasonable efficiency.

Infrared searchlight-and-detector systems can use circular polarizers with success, and the same is true of visible-light searchlight systems; Nathan (N-1) has found that when nighttime visibility is reduced by a thin fog the contrast of typical objects can be increased by a large factor by means of circular polarizers.

9.9. Coding and Decoding of Images for Stereoscopic Viewing. The old-fashioned stereoscope required no coding or decoding operations. The pictures were so small and near by that a small black vane, or septum, sufficed to prevent either eye from seeing the picture meant for the other eye. When, however, stereoscopic images are projected onto the screen of a motion-picture theater, a septum can no longer be used. The light itself must be coded in some manner. Anaglyph coding, which assigns different wavelength bands to the two images, is moderately successful; however, it produces retinal rivalry, and is, of course, inapplicable to *colored* stereoscopic pictures.

Polarizers constitute a superb solution to the problem. A linear polarizer P_R is installed at some azimuth A in front of the projector

that projects the image meant for the right eye, and a similar polarizer P_L is installed at azimuth $(A + 90°)$ in front of the projector that projects the left-eye picture. The spectator is given a pair of *three-dimension (3-D) viewers*, whose right and left windows contain polarizers aligned with P_R and P_L respectively. If the projection screen is of polarization-conserving type, and if the spectator keeps his head reasonably level, his right eye sees only the right-eye picture and his left eye sees only the left-eye picture. Accordingly, the desired stereoscopic illusion results.

As early as 1939, stereoscopic motion pictures were shown to large audiences (at the New York World's Fair) by the method just outlined. The transmission axes of the projector filters were horizontal and vertical, and the polarizers mounted in the 3-D viewers were oriented similarly.

When, in 1952, 3-D motion pictures began to enjoy a wide popularity, a new convention as to polarizer orientation was adopted. The projector for the right-eye pictures was equipped with a polarizer whose axis was at $-45°$ (that is, upward to the left, downward to the right) as judged by a person situated at the screen and looking back toward the projector. The polarizer for the other projector was mounted at plus 45°. The right and left windows of the viewers had corresponding orientations. The projector filters were usually of K-sheet, which is capable of surviving the high temperatures involved. The projection screens were of smooth, aluminized, high-gain type. The 3-D viewers usually employed HN-38 sheet.

When all the equipment was in good condition, the stereoscopic illusion was successful and highly dramatic. Often, however, conditions were poor. The two projectors often failed to stay in synchronism within the necessary tolerance (about 0.02 sec); various means for monitoring and readjusting the synchronism were developed (Jones and Shurcliff, J-31). Certain types of screens failed to have a high enough degree of polarization conservation and hence produced troublesome ghost images, according to a survey reported by Shurcliff (S-11). Some brands of 3-D viewers had far too low a polarizance and had various mechanical shortcomings also. An analysis of what constitutes a satisfactory 3-D viewer was made by Chubb, Grey, and others (C-16), who concluded that the value of k_{vx} for the individual window should be no more than about 0.002.

The same principle can, of course, be used for projecting still

pictures. Various dual-beam projectors are produced commercially, and 3-D viewers of the above-mentioned 45°–45° type are supplied with them.

Stereoscopic pairs of large-size x-ray transparency photographs can be viewed effectively with the aid of coding polarizers, a beam-combining semimirror, and a pair of 3-D viewers, as shown by Land (patent 2,084,350) and Stamm (S-26).

Of course, coding could be accomplished by means of right- and left-circular polarizers, instead of linear polarizers (Land, patent 2,099,694). Such coding would ease the tolerance on the extent to which the spectator may tilt his head to right or left without noticing ghost images. However, to produce circular polarizers that would be sufficiently achromatic would be difficult, and harmful obliquity effects might be encountered, as indicated in Sec. 9.4.

The above-described scheme for projecting three-dimensional motion pictures involves two separate film reels and two separate projectors, mounted side by side. Accordingly, the method may be called a *parallel-projection* method. (Later paragraphs discuss a series-projection method.) In about 1953 several parallel-projection schemes involving just one film reel were developed by the Synthetic Vision Corporation and the Pola-lite Corporation. The right-eye and left-eye pictures were projected from laterally distinct stations, so that the schemes were of parallel-projection type; however, all the pictures were contained on a single film, so that only one projector was required and no change in synchronism could occur. The individual frames were usually of reduced size, which reduced the clarity and brightness of the picture seen on a screen of given size. Special beam-splitter devices were used to separate the right-eye and left-eye beams sufficiently that each could be polarized independently.

Vectograph System. A conventional stereoscopic pair consists of two photographs situated side by side, that is, in parallel; consequently *two* projectors, situated side by side, must be used. It is possible, however, to mount the two members of the pair in series, so that a beam that passes through one member passes through the other also. A single projector then suffices, and questions of synchronization, registration, brightness match, and so forth, are avoided. To prevent the two images from becoming irretrievably intermingled, one must employ coding. Again, polarization coding appears almost ideal.

A practical method of accomplishing the coding was worked out in

the late 1930's by Land, Mahler, West, Rogers, Binda, and others (A-13, D-21, K-19, L-6, P-23, P-25; patents 2,204,604, 2,263,316, 2,281,101, 2,289,714, 2,289,715, 2,299,906, 2,315,373, 2,329,543). Each member of the stereoscopic pair employs as light absorber, in lieu of the silver in the ordinary photograph, aligned dichroic molecules, the pattern of which constitutes the image. In the right-eye member, all the dichroic molecules have the same direction A; in the left-eye member the dichroic molecules have the orthogonal direction $A + 90°$. Thus the two members operate in different communication channels; specifically, the polarization form that one member is capable of absorbing is transmitted freely by the other member, and vice versa. The two members have significantly different distributions of absorber, according to the shape and other characteristics of the object to be portrayed. As before, the beam strikes the projection screen and the reflected light returns toward the spectator. Before it reaches his eyes it is decoded by the 3-D viewers, and accordingly each eye receives light from just one channel. The name *vectograph* has been used to designate a series-mounted stereoscopic pair employing polarization coding. The polarization forms ordinarily assigned to the two channels are the linear forms having the azimuths $+45°$ and $-45°$ from the horizontal.

Nonuniform orientation of the dichroic dye, or use of dye molecules having a low dichroic ratio, would cause some interplay or crosstalk between the two channels. Each eye would see a faint subtractive ghost image in addition to the main image (subtractive ghosts and methods of ameliorating them are discussed in Mahler's patent 2,674,156 and Ryan's patent 2,811,893). Fortunately, dyes having high dichroic ratio are available (Sec. 4.11), and excellent orientation is achieved by methods described in Chapter 4.

If neutral dyes are used, black-and-white vectographs result. By using a suitable set of colored dyes, one can produce vectographs in full color.

It is also possible to make reflection-type vectographs. Such a vectograph contains, in addition to the (transparent) image-containing polarization-coded layers, a third layer consisting of a mat-surfaced aluminum reflector. Light that falls on such a vectograph passes through the polarization-coded layers, strikes the reflecting layer, and passes back through the polarization-coded layers. Since the reflecting layer is a polarization conserver and thus does not degrade

the coding, an observer viewing the assembly through 3-D viewers obtains the usual three-dimension illusion. Even colored reflection-type vectographs have been made. The ease of using a reflection-type vectograph is noteworthy: no projector is required, no projection screen, no special holder, no lenses.

Vectographs can be prepared from line drawings as well as from photographs. The line drawing may portray, for example, the three-dimensional structure of a complicated engineering design, and thus may help engineers grasp the design quickly and easily.

The two photographs (or drawings) can, of course, portray entirely different scenes; depending on the choice of azimuth of the analyzer, either scene may be selected by the observer. Such two-choice portrayals are suited to showing how an instrument appears before and after the housing is removed, or how a patient appears before and after treatment.

Overcoming Defects in Binocular Vision. Various training devices for helping persons with imperfect binocular vision have been used. In devices invented by Sawyer (patent 2,837,087) and by Thorburn (patent 2,837,086), two fields are provided, one for each eye; each field is incomplete, but when both eyes are performing properly, and with correct coordination, the two incomplete fields appear to combine and form one complete field. Linear polarizers are used in coding the two fields, and the patient's 3-D viewers accomplish the decoding.

Three-dimension Television. Linear polarizers are employed in several kinds of 3-D television presentation systems. In one version (A-14, A-15) the right-eye and left-eye pictures appear on two separate picture tubes; each picture is then coded by means of a linear polarizer placed in front of the picture tube. The coded beams emerging from the polarizers strike a semimirror which serves as a beam combiner; thus one combined beam results. The observer wears 3-D viewers, and accordingly each eye sees only the picture intended for it. In another version, developed by the General Electric Company in 1957, the two pictures are produced sequentially on a single picture tube, and the two coding filters are introduced sequentially and in synchronism, by means of a rotating drum that surrounds the picture tube.

Three-dimension Radar. Radar presentation, also, could employ polarization coding and could provide 3-D images. Some means for accomplishing this have been proposed by Marks (patent 2,777,011).

Three-dimension Microscopy Using a Mono-objective Instrument. Even a mono-objective microscope can be made to give 3-D images, provided the microscope has two oculars and provided a split-field polarizer is mounted just below the condenser so as to code the two halves of the beam differently. Each ocular is provided with a polarizer aligned with one half of the split-field polarizer. The method has been described by Lester and Richards (L-20); see also Kossel's patent 2,809,555.

9.10. Control of Spectral Energy Distribution. Since some of the main parameters of polarizers and retarders vary with wavelength, it is not surprising that various trains of such devices can be used to control the spectral energy distribution (or color) of a beam of light. If the train serves to vary the saturation of the color, without altering the hue, the train is called a *variable-saturation* filter. If it varies the hue, it is called a *variable-hue* filter. Four kinds of trains are of importance:

Class 1 train: a chromatic linear polarizer and an achromatic linear polarizer.

Class 2 train: two chromatic linear polarizers and an achromatic linear polarizer.

Class 3 train: two achromatic linear polarizers and one chromatic linear retarder situated between them.

Class 4 train: two or more achromatic linear polarizers and a large number of chromatic retarders.

In a *Class 1 train,* consisting of a chromatic linear polarizer P_c and an achromatic linear polarizer P_a, the chromatic behavior of the train depends, obviously, just on P_c. This consists, ordinarily, of a dichroic polarizer containing an oriented dichroic dye that absorbs in just one portion of the visual range, for example, in the region below 600 mμ, as indicated in Fig. 9.3. The k_1 and k_2 curves of P_c are high (approximately 1.0) in the wavelength region above 600 mμ; below 600 mμ, k_1 remains high and k_2 is approximately zero. Thus the absorption band is brought fully into play, or remains entirely out of play, according to whether θ, the angle of crossing of P_c and P_a, is 90° or 0°. Intermediate angles lead to intermediate amounts of absorption. The generalized law of Malus,

$$H_\theta = (H_0 - H_{90}) \cos^2 \theta + H_{90},$$

holds for each wavelength, and it is easily shown that, even for the entire visual range as a whole, a cosine-squared law applies.

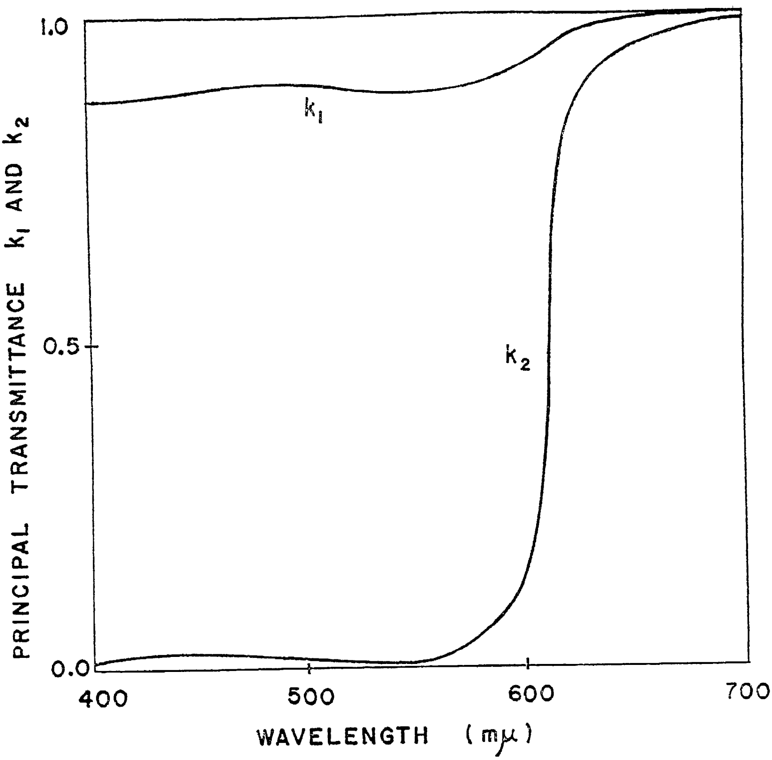


FIG. 9.3. Spectral curves of k_1 and k_2 for a typical chromatic linear polarizer containing an oriented dichroic red dye.

The Class 1 train is usually employed as variable-saturation filter. The dichroic dyes used have broad absorption bands in most instances, and consequently the train performs well where broad bands are desired. One successful application is to the control of the color of the light emitted from the screen of an airborne radar set. Usually the radar operator is glad to have the screen emit light of all wavelengths; but if (at night) he is trying to maintain his dark adaptation, he must eliminate wavelengths shorter than 600 mμ. He can do this conveniently with the aid of a train consisting of an achromatic polarizer (for example, HN-32) and a chromatic polarizer employing a dichroic red dye (see Secs. 4.9 and 4.11); by mounting the train in front of the screen, and rotating one polarizer relative to the other, he can attenuate the wavelengths shorter than 600 mμ to any extent desired, without altering the passage of longer wavelengths. The same goal

could be accomplished by placing an ordinary red filter over the screen, but that filter would have to be removed entirely in order to restore the white light; storing and remounting the red filter would be inconvenient.

Another application is in distinguishing between the (blue) prompt trace and (yellow) persistent trace appearing on certain cathode-ray-oscilloscope screens, for example, screens employing the RTMA P-7 phosphor. By turning the rotatable member of a suitably designed Class 1 train, the operator can block the blue light or allow it to pass, and thus can concentrate on whichever type of trace he deems most significant. Another application is to safelights used in photographic darkrooms; a Class 1 train can be designed to block or pass actinic light, depending on the setting of the rotatable member of the train; nonactinic light is transmitted freely at all times. Ryan, in patent 2,263,684, has described a train well suited to darkroom use.

The *Class 2 train* consists of two orthogonally oriented chromatic polarizers P_{c1} and P_{c2} and an achromatic polarizer P_a. Usually P_{c1} and P_{c2} have absorption bands lying in adjacent portions M and N of the visual range; also, they are usually bonded together with their transmission axes at 90° to one another. Thus the azimuth of the third element, P_a, determines which of the portions M and N is transmitted. Such a train constitutes, obviously, a variable-hue filter. Again, the bands are broad, and the applicability of the filter is limited accordingly. A version of this filter marketed by the General Electric Company has been used successfully in altering the blueness-redness ratio of the light entering the lens of a color camera (D-12, G-4). In this device P_{c1} has an absorption band in the short-wavelength half of the visual spectrum and P_{c2} has an absorption band in the long-wavelength half. When P_a is at 45°, both bands are attenuated moderately; when P_a is at 0°, one band is absorbed strongly and the other is absorbed very little; when P_a is at 90°, the situation is reversed. Intermediate settings lead, of course, to intermediate color balances. Obviously, such a continuously adjustable filter can take the place of an entire graduated series of filters of conventional type.

The *Class 3 train,* employing a chromatic linear retarder between two achromatic linear polarizers, is of relatively little importance. The retarder's retardance varies only slowly with wavelength, and hence the absorption bands that it leads to are broad and ill defined. Also, most Class 3 trains suffer from having large obliquity effects: rays

incident at different angles from the normal experience different retardance, hence emerge differently colored. However, a wide range of colors can be produced, and can be used successfully in advertising displays. Particularly striking effects are produced if the retarder consists of a mosaic of regions having different ρ and δ, and if one of the polarizers is rotated slowly and continuously. Each part of the mosaic passes through a sequence of brilliant colors; thus a constantly changing color pattern results (Land's patents 2,146,962 and 2,158,129). An interesting Class 3 train has been described by Stadler (patent 2,527,593); his retarder employs two layers of special type, and produces an unusually wide range of colors.

The *Class 4 train*, which includes a large number of retarders, is the most interesting class. Certain designs have an almost monochromatic pass band and thus may be used by astrophysicists in taking photographs of solar prominences by means of light of 6563 A, where hydrogen has a prominent spectrum line. Designs by Lyot, Öhman, and Šolc have attracted much attention.

The Lyot filter, invented in 1933 by the young French astronomer Bernard Lyot, employs an alternating series of polarizers and retarders, and succeeds in having a pass band only 1 to 5 A in width. The design has been described clearly in the literature (A-7, L-28, L-29, O-1, E-16, E-17), and will merely be summarized here. In a typical design, six neutral linear polarizers with transmission axes at 45° are employed. Five linear chromatic retarders with fast axes horizontal are interspersed between the polarizers. The first stage, consisting of the first retarder and its flanking polarizers, produces for the desired wavelength of 6563 A a retardance of 360°, so that the transmittance of this stage for this wavelength is very large; other wavelengths are retarded to different extents, and are transmitted to lesser extent. Each retarder (a principal section of quartz, for example) has twice the thickness of the preceding retarder, and thus has twice the retardance; hence each retarder provides twice the wavelength discrimination that the preceding retarder provides. Consequently the only light that passes through the entire train is light whose wavelength is almost exactly 6563 A. (Other pass bands may occur in remote parts of the spectrum, but are eliminated by conventional absorption or interference filters.) The over-all transmittance of the train for 6563 A is of the order of 10 percent. The permissible angular aperture is about 2°. The assembly may be 10 or 15 inches long. The

pass band may shift appreciably from time to time unless the temperature of the train is held constant within 0.01°C. The expense of such filters is so great that few units have been constructed. A number have been made by Baird-Atomic, Inc., of Cambridge, Massachusetts, and by Optique et Précision de Levallois of Paris, France.

A somewhat similar design was evolved independently by Öhman (O-1). This design employs dichroic, rather than birefringence, polarizers, and hence is more compact; unsupported K-sheet has been found to perform very satisfactorily. A design proposed by Billings (B-1, B-21) contains provision for adjusting the transmitted wavelength at will through a range of a few angstroms. Dollfus has experimented with a device that has a pass band centered near 1.4 μ and is intended for use (in conjunction with a lead sulfide detector) in studying the atmospheres of various planets. Steel, Smartt, and Giovanelli (S-26a) have designed a device having a pass band only $\frac{1}{8}$ A wide.

In 1953–1955 the Czechoslovakian scientist Ivan Šolc (S-22, S-23, E-17) invented a kind of narrow-pass-band filter that has a surprisingly high transmittance. The Šolc filter consists of two achromatic linear polarizers between which is a stack of 60 identical linearly retarding (chromatic) layers each with its axis shifted slightly relative to that of the preceding layer. Again the result is that just one narrow band of wavelengths is transmitted efficiently and all other wavelengths are absorbed strongly. Evans has shown that the performance to be expected of such a filter can be calculated easily with the aid of the Jones calculus (Chapter 8). He made a careful comparison (E-17) of the Lyot filter and the Šolc filter, and concluded that the latter, though having much greater over-all transmittance, suffers from low spectral purity.

Both the Lyot filter and the Šolc filter are encountering increasing competition by interference filters, such as that recently described by Dobrowolski (D-13). His filter provides a pass band as narrow as 1.0 A with a transmittance of approximately 50 percent.

Certain Class 4 trains may serve as variable monochromators. A train containing several achromatic linear polarizers and intervening (chromatic) circular retarders of graduated retardance may have a pass band whose central wavelength can be varied over a wide range by appropriate rotation of the polarizers. A variable monochromator of this type (employing four circular retarders consisting of basal sections of quartz) has been described by Hurlbut and Rosenfeld (H-39)

and in Hurlbut's patent 2,742,818, and a monochromator of somewhat similar type has been produced commercially by the Cambridge Thermionic Corporation of Cambridge, Massachusetts. Lyot, in patent 2,718,170, has described a spectrophotometer that employs polarizers and retarders instead of prisms or gratings; the spectrophotometer contains no slits.

Land (patent 2,493,200) has proposed a design of a Class 4 train that employs several spectrally selective dichroic polarizers and several electro-optically controlled retarders. By adjusting the electric field governing the retarders, one can alter the wavelength band, and hence the color, of the transmitted light. The alterations can be accomplished very quickly; consequently the train could be used to vary the color of the light emitted by a television picture tube. Successive fields could be caused to appear in different colors, so that a field-sequential type of color-television presentation could be achieved. (See also Sage's patent 2,834,254.)

Channeled Spectra. The combination of a linear retarder sandwiched between two aligned linear polarizers produces what is known as channeled spectra (D-10). If the retarder is of chromatic type, has a large retardance, and is mounted at 45° to the axes of the polarizers, certain wavelengths will be unable to pass through the second polarizer and certain other wavelengths will pass through freely. If the emerging beam is dispersed by a prism, so that a broad spectrum results, certain portions (the channels) will appear blank and certain other portions will appear bright. The greater the retarder's retardance, the greater the number of channels. If one measures the central wavelengths of the channels, one can easily determine the values of retardance and planobirefringence at these wavelengths. The voluminous literature on channeled spectra has been surveyed by Jerrard (J-13); see also Ellis and Glatt (E-14).

10 OTHER APPLICATIONS TO SCIENCE AND TECHNOLOGY

10.1. Introduction. This chapter deals with the application of polarizers to the appraisal of specimens which themselves exhibit polarizance or retardance. It deals, in other words, with applications in which polarization is of interest from the outset. The applications are closely tied to chemistry, crystallography, metallurgy, physics, electronics, biology, and astrophysics.

10.2. Vision and the Haidinger-Brush Phenomenon. In 1844 Haidinger (H-6) found that the polarization of light can sometimes be detected directly by eye. He discovered that if an observer gazes for a few seconds at a clear field of white light that is linearly polarized with the electric vibration horizontal, then glances at a clear field in which the vibration direction is *vertical*, a faint pattern is seen. As indicated in Fig. 10.1, the pattern consists of a small yellowish brush — actually a double-ended brush — with intervening areas that appear bluish.

If the observer now glances back at the horizontally polarized field, he sees a similar pattern except that the brush axis is now vertical. The patterns appear to fade out within a few seconds, and the higher the level of illumination the more rapid the fading. Stokes (S-28) found that if the fields are lacking in blue light no brush is seen. Conversely, Sloan (S-18) found that if the field consists solely of blue light the contrast of the brush is accentuated. (Some observers fail to see the brush even when conditions are ideal.)

Shurcliff (S-13) found that *circularly* polarized fields, also, tend to

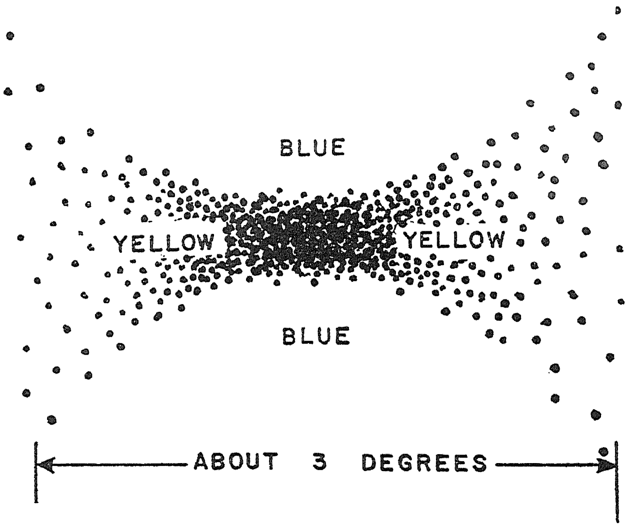


FIG. 10.1. Typical appearance of Haidinger's brush when the field of linearly polarized white light has a *vertical* vibration direction.

produce brushes, and that right- and left-circularly polarized light produce brushes at different and distinctive azimuths, so that — merely by eye — an observer can tell the handedness of polarization. Right-circularly polarized light produces (for most observers) a brush that has an upward-to-the-right, downward-to-the-left direction (assuming that the observer holds his head vertical). Left-circularly polarized light produces a brush having the orthogonal direction.

The brushes presumably betoken (*a*) linear dichroism in the yellow pigment of the macula of the eye, and (*b*) birefringence effects in the eye's refracting media. See also articles by Helmholtz (H-19), de Vries and others (D-8), Boehm (B-38), and Cords (C-28).

10.3. Polarization and Navigation. In about 1949 the National Bureau of Standards, in carrying forward some work started by A. H. Pfund, perfected a polarization-sensitive *sky compass* intended for daytime use in regions near one of the earth's magnetic poles, that is, regions where magnetic compasses perform poorly. The device is designed (A-12, P-24) for use by persons who are not equipped with electronic direction finders and are unable to see the sun directly, as when there is moderately extensive cloud cover or the sun is slightly below the horizon. The device takes advantage of the fact that the

predominant vibration direction of light from the blue sky tends to be perpendicular to the plane containing the sun and the observer's line of sight. By finding the predominant vibration direction of light from, say, the zenith, and taking into account the time of day, the direction of true north can be estimated.

In 1953 Campbell (C-2) described a modified device intended to assist geologists in certain situations. An accuracy of about 2° is claimed.

In the 1940's Frisch (F-17) found that bees can distinguish the polarization of light and can control their flight path in relation to the dominant vibration direction. Thus they succeed in returning repeatedly to a known source of honey even when the sun is obscured and only a small region of blue sky is visible. Autrum and Stumpf (A-23), Kennedy and Baylor (K-7), and Thorpe (T-6) also have studied bees' ability to navigate in this way.

Ants, too, appear to take account of the polarization of light from the sky. The same applies to the horseshoe crab *Limulus*, studied by Waterman (W-5, W-6), the crustacean *Talitrus saltator* (K-1, K-2), the fruit fly *Drosophila*, and the water flea *Daphnia* (W-6). See also articles by Baylor and Smith (B-11), Waterman (W-8), Jander and Waterman (J-4), Wellington (W-15), and Waterman and Westell (W-7).

10.4. Production of Biological Effects. In 1956 Jaffe (J-1) showed that when the (spherical) fertilized egg cells of the brown alga *Fucus* are exposed to linearly polarized light, and when, as a consequence, they later begin to grow, the growth tends to be parallel to the direction of the electric vibration of the light. See also J-2 and W-15.

Schöne *et al.* (S-6) have shown that polarized light plays a significant role in controlling the movements of the eyestalks of the crab *Ocypode quadrata*.

10.5. Polarimetry. To some physicists the term *polarimeter* means a device for determining the azimuth or ellipticity of a beam of polarized light. To chemists, the term usually means a device for measuring the *change* in azimuth of a beam of linearly polarized light, the change being produced when the light passes through a circularly birefringent solution (that is, an *optically active* solution). Having determined the change in azimuth, the chemist can usually compute the concentration

C of the circularly birefringent solute in the solution. Thus, for example, he can determine the amount of dextrose in an aqueous solution. If the change in azimuth is called ζ (known as the rotation angle; Sec. 7.5) and if the concentration C is expressed in terms of grams of solute per milliliter of solution,

$$C = \frac{\zeta}{L[\alpha]},$$

where L is the path length (in centimeters) and $[\alpha]$ is the *rotatory power*. Since the rotatory power varies greatly with the frequency of the light (often being proportional to the square of the frequency), monochromatic light must be used if precise values of rotation angle are to be attained.

Polarimetry is a mature technology and has been discussed in many books, including those by Heller (H-18), Bates (B-6), Browne and Zerban (B-52), and Bruhat (B-54). The simplest polarimeter consists essentially of a monochromatic light source and two linear polarizers; the polarizers are initially in crossed orientation, and thus extinguish the light. The specimen is placed between the polarizers, and the operator's task is to measure the amount by which one of the polarizers must be rotated in order that the light may again be extinguished. The specimen itself is usually a liquid solution confined in a container whose windows consist of nonbirefringent glass mounted in stress-free manner. The precision of determining the rotation angle is increased if a split-field system is employed; the operator's task is then to adjust the rotatable polarizer until both portions of the split field appear equally dark. To split the field, one may insert a small birefringence polarizer, called a Lippich prism, in one half of the field; the prism is tilted slightly to right or left, so that the vibration direction of the light emerging from it differs by a few degrees from that of the remainder of the field. As explained by Rudolph (R-16) and other authors (I-7, K-13), the optimum extent of tilt depends on the brightness of the source and the quality of the polarizers. If the brightness is great and the polarizers have extremely small H_{90} values, a tilt angle as small as $2°$ may be optimum. In other situations a tilt of $5°$ or even $10°$ may be preferable; see I-7. Because it is important that H_{90} be small, birefringence polarizers or HN-22 polarizers are especially appropriate. (Other split-field devices are discussed in Sec. 7.11.)

Several recent types of polarimeters employ photocells, instead of the eye, as detector, and afford increased precision, such as 0.01° or even 0.001° (R-16, H-44). Also, they may be capable of working at wavelengths lying outside the visual range (Mitchell, M-22, and Duverney and Vignoux, D-25). In some instruments two photocells are used, one for each half of the split field; in other instruments a single photocell scans the two fields sequentially. Designs employing two photocells have been described by Downie (D-15) and by Keston (patent 2,829,555). Designs employing a single photocell have been produced by Rudolph (R-16), Malcolm and Elliott (M-9), Hyde *et al.* (H-44), and Brode and Jones (B-49). In some instances polarimeters have been teamed with variable monochromators, to yield *spectro-polarimeters;* see Billardon and Badoz (B-20), Klyne (K-11), Hardy (H-12), Mitchell (M-22), and Mitchell and Veitch (M-21). A polarimeter designed for studying eclipses has been described by Ney *et al.* (N-3).

Takasaki (T-2) has described a polarimeter that employs a Senarmont compensator in which an ADP (ammonium dihydrogen phosphate) crystal excited by alternating current is incorporated.

Instruments designed specifically for measuring the concentration of sugar solutions are known as saccharimeters; these have been discussed at length by Bates (B-6) and Browne and Zerban (B-52). Polarimeters especially designed for use with gases, as in measuring the Faraday effect of such gases, have been described by Ingersoll and Liebenberg (I-1, I-2); pathlengths as great as 20 meters are provided. A polarimeter design applicable to radio microwaves has been described by Allen (A-3); see also Cohen (C-18). A detailed discussion of the Faraday effect has been presented by Waring and Custer (W-4).

Cause of Circular Birefringence. Much has been written on the theory of circular birefringence. Perhaps the principal works are those of Born (B-42), Chandrasekhar (C-9), Condon (C-23), Brode (B-50), Lowry (L-27), and Federov (F-8). Tables of rotation constants of many materials have been published by Malleman (M-11), Ingersoll and Liebenberg (I-3), and many others. Much of the work done by Malleman and by Ingersoll and Liebenberg has been concerned with gases that exhibit circular birefringence when subjected to a magnetic field, that is, gases exhibiting *Faraday effect.*

10.6. Microscopy. The design of polarizing microscopes has been discussed by many investigators, including Ambronn and Frey (A-5), Burri (B-57), Hallimond (H-10, H-11), Hartshorne and Stuart (H-16), Johannsen (J-18), Rinne and Berek (R-9), Swann and Mitchison (S-33), Vickers (V-4), and West (W-17). Applications to metallography have been surveyed by Dunsmuir (D-23) and by Conn and Bradshaw (C-24); applications to ceramics have been discussed by Insley and Frechette (I-8). A typical polarizing microscope employs two polarizers, one situated beneath the condensing lens and the other (the analyzer) situated above the objective lens; at these locations the rays are approximately parallel to the microscope axis, and consequently obliquity effects such as have been discussed by Baxter (B-7) are minimized. Ahrens polarizers are often used (Sec. 5.4); recently the HN-22 dichroic polarizer has been used extensively. In appraising the polarization form of the light that has passed through the specimen, the investigator may employ an adjustable retarder, called a compensator (Sec. 7.11).

In recent years the polarizing microscope has been improved greatly by Inoué (I-4) and by Inoué and Hyde (I-5, I-6). By rigorously excluding extraneous birefringence effects such as are often exhibited by condenser and objective, and by introducing *polarization rectifiers*, they made a large increase in sensitivity and contrast. The purpose of the rectifier is to offset, or annul, the tendency of peripheral portions of the condenser or of the objective to alter the vibration direction of the linearly polarized light striking these portions. Since the lens is deeply curved, rays striking it at different distances from the center and at different azimuths are subjected to different polarization-by-reflection tendencies; consequently, if the light incident on the lens has a single vibration direction, the light that emerges from the lens will inevitably display a variety of vibration directions. As a result, no one angular position of the analyzer can extinguish all of the rays associated with a given portion of the specimen, and small birefringence effects in the specimen will escape notice. The rectifier, also, employs curved surfaces, and hence produces a variety of changes in polarization form; but it employs in addition a 180° linear retarder at such an azimuth as to reverse the changes in vibration direction, so as to compensate exactly for the changes produced by the adjacent lens. When one such rectifier is used just below the condenser and another is used just above the objective, excellent extinction can be achieved

(I-5) even when the numerical aperture of the objective is as great as 1.25. Thus it becomes possible to detect retardances as small as 10^{-3} cycles even in, say, intracellular objects only 1 μ in diameter. Using this approach, Inoué has succeeded in seeing and photographing intracellular birefringence patterns and nuclear organizations not detectable by any other means.

Phase-Contrast Microscopes. Several investigators have attempted to improve the usual phase-contrast microscope by employing polarizers that serve to (*a*) provide control over the degree of contrast or (*b*) provide a different degree of contrast for different wavelengths so as to achieve variable-color phase contrast. See an article by Barer (B-2).

Interference Microscopes. The interference microscope employs two coherent beams; one passes through the specimen, and the other does not; the two beams are then recombined. The pattern that the observer sees is indicative of the variations in optical pathlength within the specimen. In several designs of interference microscopes, polarizers play central roles: they are used in producing the initial, coherent (linearly polarized) beam, and in selecting (from the recombined beam) energy having one vibration direction. Also, polarizing beam splitters and polarizing beam combiners may be used. See articles by Smith (S-20), Barer (B-4), Françon (F-14), Koester (K-13, K-14), Inoué and Koester (I-7), and Richards (R-6).

10.7. Crystallography and Chemical Structure. Many crystals and many oriented polymeric materials exhibit retardance and dichroitance with respect to ultraviolet, visible, or infrared light. By measuring the retardance and dichroitance (and computing the intensive parameters birefringence and dichroism), and by noting the axis directions, an investigator may be able to identify the material in question. If the specimen is of new type, his measurements may help him determine its chemical structure.

If the specimen is very small, or if the absorption per unit pathlength is very great, best results may be obtained with the aid of the polarizing microscope, discussed in the preceding section. If the specimen is of ample size and is homogeneous, precise information as to dichroism may be obtained by means of a spectrophotometer equipped with a polarizer.

The procedures used in studying the birefringence of crystals have

been described in references given in the preceding section. In addition, monographs by Försterling (F-12), Born (B-42), Bouasse (B-44), and Pockels (P-19) may be consulted; see also Dreyfus (D-19). The study of birefringence of organic and biological materials has been surveyed by Bennett (B-13), Gurnee (G-14), Oster and Pollister (O-5), Stein (S-27), Locquin (L-25), Schmidt (S-5), and Spence (S-25); interesting applications to the living cell have been described by Inoué and others (I-4, I-5, I-6).

Treatises on circular birefringence have been listed in Sec. 10.5; see also Jones (J-20) and Pancharatnam (P-3, P-5). The circular birefringence of liquid crystals (for example, cholesteryl acetate and cholesteryl benzoate) has been discussed by Lawrence (L-16). Circular birefringence caused by absorption of circularly polarized light has been discussed by Zocher and Jacoby (Z-9).

The experimental methods used in measuring dichroism have been discussed by West and Jones (W-22), Charney (C-13), and various others (B-8, B-28, B-34, O-5). Applications to dichroic dyes and microcrystals have been described by Land and West (L-8), Land (L-13), Ambrose *et al.* (A-6), Blout *et al.* (B-34), Bovis (B-47), Conroy (C-27), Lambe and Compton (L-3), LeRoux (L-19), and Scherer (S-3). Applications to organic films and fibers have been discussed by Ambronn (A-4), Ambronn and Frey (A-5), Blout and Karplus (B-32), Blout *et al.* (B-33), Blout and Bird (B-35), Elliott (E-11, E-12), and Krimm *et al.* (K-20). Circular dichroism has been treated by Bruhat (B-53, B-55); see also Cotton (C-29).

The subject of birefringence of flow (also called double refraction of flow, or streaming birefringence) has recently come into prominence. The procedures have been described by Scheraga and Signer (S-2), Boehm and Signer (B-37), Rich (R-5), Muralt and Edsall (M-29), Edsall *et al.* (E-6), Peterlin and Stuart (P-14), Cerf and Scheraga (C-6), Perry (P-13), Janeschitz-Kriegl (J-5), Zimm (Z-4), and Lodge (L-23). Even when the flow exhibits some turbulence (rather than being steady), some success can be achieved, as explained by Wayland (W-9).

Flow dichroism, also, has come into prominence. Again, shearing flow is used as a means of aligning the molecules; and while the flow is maintained the dichroism of the liquid is measured. The method has been discussed by Zocher and Jacoby (Z-6) and by Bird *et al.* (B-26).

10.8. Photoelastic Analysis. Photoelastic analysis is the technology that relates retardance and mechanical strain. If a flat strip of glass is subjected to a steady longitudinal tension, the glass becomes slightly elongated (strained); as a consequence it becomes birefringent and exhibits retardance. Provided the strain is small, the retardance is proportional to the strain; for many materials the proportionality constant (the stress-optic constant) is known. Accordingly, by measuring the retardance one may compute the magnitude of the strain. The directions of the fast and slow axes indicate the principal axes of strain.

In the classical exploitation of photoelastic analysis, the entire specimen consists of a transparent material that has a high stress-optic constant (H-25, B-10). The equipment used and the procedures involved have been described in detail in several books, including those by Jessop (J-17) and by Coker *et al.* (C-20); other important references are F-9, A-2, F-18, H-25, and J-15. The heart of the equipment is the *polariscope*, which consists of an illumination system, a pair of polarizers, and means for holding the specimen in position between the polarizers. Many types of polariscopes are produced commercially. Some employ monochromatic sources and enjoy advantages as to precision, ease of photoelectric measurement, and convenience of recording by means of black-and-white photography. Other types employ white light and produce colored patterns; the colored patterns yield more information and often permit the investigator to determine the number of cycles of retardance directly by eye, provided the number is low. Most polariscopes employ linear polarizers, and are well suited to determining the principal axes of strain; since the polarizers are virtually achromatic, broad bands of wavelengths may be used successfully. Polariscopes that employ circular polarizers are well suited to determinations of the *difference* between the principal strains; however, circular polarizers are usually circular for the middle of the visual range only, and perform imperfectly for other parts of the visual range.

The classical method of photoelastic analysis suffers from one large drawback: the specimen must be transparent. The majority of objects of interest to mechanical engineers are, of course, opaque. To see what patterns of strain are produced in such a body, the investigator must first construct a model out of a transparent material; the desired stress system is applied to the model, and the resulting strains are

determined. To prepare such models is time consuming, and doubts may arise as to the comparability of the behavior of the model and behavior of the original object.

A new approach, applicable even to opaque specimens, has been developed in recent years; this approach is often called the *photostress method*, but might more appropriately be called "indirect photoelastic analysis by means of an adhering film." The essence of the method, proposed by Mesnager (M-18) and elaborated by D'Agostino (D-1, D-2) and by Zandman and Wood (Z-1, Z-2), is to apply a reflecting coating to the specimen and then superimpose on this a transparent coating consisting of a material having a large stress-optic constant. The specimen is then illuminated with a parallel beam of polarized light, and the specularly reflected light (which has passed twice through the transparent coating) is viewed through a polarizer. When the specimen is strained, the transparent coating becomes strained also, and by evaluating these strains one infers the strains existing in the surface layers of the specimen proper. In general, the method is successful; however, D'Agostino (D-1) has warned against errors that may arise as a result of nonlinearity between strain and birefringence or as a result of creep of the adhering film.

Various special-purpose polariscopes have been constructed. An infrared polariscope has been developed by Appel and Pontarelli (A-17); Glan-Thompson polarizers and quartz retarders are used, and a photographic film sensitive to wavelengths as great as 1.2 μ is used as detector. Besse and Desvignes (B-17) have described an infrared polariscope employing obliquely mounted silicon mirrors as polarizers. So-called three-dimensional photoelastic analysis has been accomplished by several investigators, including Leven (L-21), Frocht (F-18), and Frocht and Guernsey (F-19). A circular polariscope designed by Mindlin (M-20) uses just one polarizer and a mirror; the mirror causes the light to pass through the specimen twice, and permits the single polarizer to serve in two capacities: as both polarizer and analyzer.

10.9. Other Applications. Hundreds of additional types of applications of polarizers and polarized light could be mentioned. Some of the more important ones are listed below:

Studying polarization phenomena in the radiation from distant stars and nebulae: Cohen (C-18), DuFay (D-22), Hall and Mikesell

(H-8, H-9), Hiltner (H-30, H-30a, H-31), Piddington (P-16), Shklovsky (S-10).

Studying the polarizing tendency of galactic dust: Chandrasekhar (C-7), DuFay (D-22), Hiltner (H-26, H-27, H-28, H-29, H-30).

Studying the amount of atmosphere on planets: Anonymous (A-16), Dollfus (D-14), de Vaucouleurs (D-6).

Mapping the magnetic fields on the sun's disk: Simpson (S-16).

Determining the polarization of light from the daytime sky: Bennett *et al.* (B-12), Chandrasekhar and Elbert (C-10), Chapman (C-12), Coulson *et al.* (C-31), Dietze (D-9), Foitzik (F-11), Jensen (J-10), Lenoble (L-18), Pollak and Wilhelm (P-28), Richartz (R-8), Sekera (S-8, S-9).

Determining the polarization of light at sea: Hulburt (H-36).

Determining the polarization of light present under water: Ivanoff (I-9), Waterman (W-5, W-6).

Monitoring the orientation of atoms and the precession of atoms; optical pumping: Culver (C-32), Dehmelt (D-4).

Detecting polarization of x-ray and gamma-ray photons: Fagg and Hanna (F-1), Jamnik and Axel (J-3), Schick *et al.* (S-4).

Measuring the torque exerted by circularly polarized light: Beth (B-18), Henriot (H-21), Holbourn (H-32).

Demonstrating nonreciprocal birefringence of ferrites: Fox *et al.* (F-13), Townes (T-8).

Detecting dislocations in crystals: Bond and Andrus (B-40).

Determining the direction of ejection of electrons by high-energy photons: McMaster and Hereford (M-4).

Determining the polarization dependence of photoresponsivity of photoconductive materials: Kelly (K-6).

Determining the dichroism of F and M absorption centers in alkali halide crystals: Compton (C-22), Van Doorn and Haven (V-2).

Studying Weigert effect and photodichroism: Helwich (H-20), Kondo (K-15), Nikitine (N-6, N-7), Weigert (W-13), Zocher and Coper (Z-8).

Determining the decay time of fluorescence: Perrin (P-11), Ravilious *et al.* (R-4).

Determining the optical constants of metals: Ditchburn (D-11), Prishivalko (P-31), Roberts (R-10).

Aiming with the aid of the optical ring sight: Land (patents 2,420,252 and 2,420,253), West (patent 2,420,273).

Measuring the straightness of a long line: Dyson (D-26).

Measuring the thickness of a very thin dielectric film by means of the ellipsometer: Rothen (R-14, R-15), Rudolph (R-16).

Measuring the diameter of a refractoanisotropic fiber: Bridges and Roehrig (patent 2,824,488), Roehrig (patent 2,824,487).

Determining the size distribution of particles: Kerker (K-8).

Secret communication system: Land (patent 2,362,832), Mueller (patent 2,707,749).

Producing harmonic musical notes: Land and Grabau (patent 2,376,493).

APPENDIX 1 — DIRECT PRODUCTION OF POLARIZED LIGHT

There are at least ten ways of producing polarized light directly, that is, without first producing unpolarized light and then interposing a polarizer. These direct methods are of interest to theoreticians, but of little practical importance.

Grazing Emergence. In 1926 Worthing (W-27) found that the light that emerges almost at grazing angle from a flat, hot (3000°C) sheet of tungsten is linearly polarized to the extent of 90 to 95 percent. The predominant direction of the electric vibration is perpendicular to the direction of propagation and approximately perpendicular to the sheet also. Much of the light presumably originates at an appreciable distance below the surface of the sheet and, in the act of emerging, suffers reflection losses; the losses are greatest, of course, for the component whose vibration direction is parallel to the surface.

Even if the hot tungsten body has the shape of a wire, the effective average polarization is appreciable; it amounts typically to about 20 percent for the light reaching a given observer from the various portions of the wire's surface. Goetze (G-9) showed that the 20-percent figure drops to about 5 percent if the wire is cooled until it appears dull red in color. Polarization from incandescent wires of other metals has been measured by Schubert (S-7). Additional work has been reported by Keussler and Mannog (K-10).

Biemissivity. If a dichroic material (for example, tourmaline) is heated to incandescence, the emitted light is rich in the polarization form of the type that the material absorbs most strongly when employed as a receiver. This behavior is, of course, in accord with

Kirchhoff's law, which associates strong absorption of some given form with strong emission of that form.

Bifluorescence. Many crystalline materials, when excited by (unpolarized) ultraviolet radiation, emit polarized fluorescent light. This is true, for example, of crystals of anthracene, chrysene, and phenanthrene, according to Ganguly *et al.* (G-3); see also Choudhuri (C-15) and Weber (W-10). By measuring the intensities of the polarized components propagated in different directions within the crystal, an investigator may learn much about the orientation of the molecule and the nature of the oscillator responsible for the emission of the fluorescent light (G-3).

Stark Effect. If an arc lamp containing, say, hydrogen is operated in a region pervaded by a strong electric field **F**, the light emitted in directions perpendicular to the field is found to be polarized. Each spectral line is split by the field into several lines, and these exhibit polarization. Light emitted perpendicular to **F** consists of lines that are linearly polarized with the electric vibration either parallel to **F** (the p component) or perpendicular to it (the s component); this is called the transverse Stark effect. Light emitted parallel to **F** is unpolarized.

Zeeman Effect. If a discharge tube containing, say, sodium vapor is placed in a strong magnetic field **H**, a typical spectral line is split into several lines. Light emitted perpendicular to **H** is linearly polarized with vibration direction parallel to **H** (p component) or perpendicular to **H** (s component); this is the transverse Zeeman effect. Light emitted parallel to **H** is circularly polarized; this is the longitudinal Zeeman effect.

Even microwaves display the Zeeman effect, as has been shown by Eshbach *et al.* (E-15).

Čerenkov Effect. When electrons traveling at nearly 3×10^{10} cm/sec strike a plate of plastic, their speed in the plastic is (temporarily) greater than that at which light travels in this medium. As a consequence visible light is emitted. The mechanism has been described in detail by Jelley (J-8); see also Linhart (L-22). The wavefront W of the light is conical and each ray is found to be linearly polarized, the direction of its electric vibration being parallel to the pertinent element of W.

Grating Plus Electron Beam. In this method, invented in 1953 by

Smith and Purcell (S-21), a stream of high-speed (300 kev) electrons is caused to pass very close in front of the face of a plane aluminized grating having, say, 15,000 grooves per inch. The stream is parallel to the plane of the grating, and is perpendicular to the grooves. As the negative charges (electrons) pass in front of the grating, positive charges are induced on the surface of the grating and move along so as to keep abreast of the electrons; in doing so, they travel up and down the ridges between the grooves. The up-and-down motion occurs at a very rapid rate, and necessarily results in the emission of electromagnetic radiation. The radiation is polarized with its electric vibration perpendicular to the grooves. The effect bears a resemblance to the Čerenkov effect, as Jelley (J-8) has pointed out.

Undulator. In this method, invented by Motz (M-25) in 1954, bunches of 2-Mev electrons are directed along a path between a succession of oppositely arranged pairs of magnetic poles. Each electron is urged laterally when passing between a pair of poles, and the successive lateral directions are in opposite sense. Thus the electron is caused to follow a quasi-sinusoidal path. Consequently it radiates linearly polarized radiation. Even if the successive pairs of poles are not located extremely close together, the resulting radiation will appear to a "downstream" observer to have very high frequency, thanks to Doppler effect and the very high speed of the electrons (about 99.99 percent of the speed of light). Thus the radiation may be of sufficiently high frequency to be visible to this observer.

Other Methods. Light from canal rays and certain other types of gaseous discharge is often polarized. There is an enormous literature of this subject. See, for example, an article by Angenetter and Verleger (A-9).

Light emitted in the Raman effect is often polarized. Here, too, there is an extensive literature. See, for example, Bhagavantam (B-19) and Cabannes and Daure (C-1).

Radiation from a maser or a laser may be polarized if the device contains a birefringent crystal or a birefringent coating on a reflecting surface; see S-1.

Radio transmitter antennas commonly radiate polarized waves (F-16). Usually the polarization is linear, but when a helical antenna, or a special arrangement of dipoles, is used circular polarization is achieved (C-14). Polarized microwaves, also, are easily produced, and

kits for demonstrating the polarization are available (C-5); see also Hull (H-38). Kaufman (K-4) has generated polarized wave trains whose wavelength is as short as 0.1 to 1.0 mm.

High-energy gamma-ray photons emitted as a result of K-capture by certain nuclides are found to be circularly polarized; see Hartwig and Schopper (H-17). Nearly 100-percent linear polarization is found in the 5- to 10-Mev gamma radiation from the $^2H(p, \gamma)^3He$ reaction, according to Wilkinson (W-25). Approximately 20-percent polarization has been found in 15-Mev bremsstrahlung radiation produced by 25-Mev electrons, according to Jamnik and Axel (J-3). Electron synchrotrons emit light that has a high degree of polarization, the vibration direction being parallel to the plane of the orbit.

Light from certain parts of the Crab nebula and some other nebulae is polarized to a considerable extent; see Sec. 10.9.

APPENDIX 2 STANDARD MUELLER MATRICES AND JONES MATRICES

The method of determining the standard matrices of the Mueller calculus and the manner of using the matrices have been discussed in Chapter 8. The same applies to the standard matrices of the Jones calculus.

The commonly used matrices are listed in the present appendix. Obviously, the exact forms of the matrices (for example, the signs of terms involving the sine of an angle) depend on the conventions used in describing the light, the polarizers, and the retarders. Accordingly, the forms listed below are valid only with respect to the conventions used in this book. See Chapters 1, 2, 3, and 7.

Abbreviations. The symbols C_1 and S_1 stand for $\cos\theta$ and $\sin\theta$ for polarizers, and for $\cos\rho$ and $\sin\rho$ for retarders. The symbols C_2 and S_2 stand for $\cos(2\theta)$ and $\sin(2\theta)$ for polarizers and for $\cos(2\rho)$ and $\sin(2\rho)$ for retarders. Also,

$$
\begin{aligned}
D &= M\sin\tfrac{1}{2}\delta,\\
E &= C\sin\tfrac{1}{2}\delta,\\
F &= S\sin\tfrac{1}{2}\delta,\\
G &= \cos\tfrac{1}{2}\delta,
\end{aligned}
$$

where M, C, and S are the second, third, and fourth Stokes parameters of the normalized fast eigenvector of a given retarder, and δ (radians or degrees) is the retardance. Finally,

$$
\begin{aligned}
Y &= \cos(2\arctan b/a), &\qquad P &= e^{i\pi/4},\\
Z &= \pm\sin(2\arctan b/a), &\qquad Q &= e^{-i\pi/4}.
\end{aligned}
$$

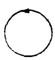
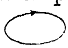
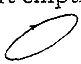
In the expression for Z the positive or negative sign applies according as the sense of the sectional pattern of the major eigenvector is clockwise or counterclockwise.

Standard Matrices. The standard matrices of miscellaneous devices, polarizers, and retarders are listed below.

Optical device	Mueller matrix	Jones matrix
Miscellaneous devices		
Region devoid of matter; or ideal plate of isotropic, nonabsorbing glass	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Ideal plate of isotropic, absorbing glass whose transmittance is k or p^2	$\begin{bmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & k \end{bmatrix}$	$\begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$
Totally absorbing plate	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Ideal depolarizer (nothing closely resembling such a device exists)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	None
Ideal homogeneous linear polarizer		
Azimuth θ of transmission axis		
0°	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
90°	$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
45°	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$-45°$ 	$\frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
General	$\frac{1}{2}\begin{bmatrix} 1 & C_2 & S_2 & 0 \\ C_2 & C_2^2 & C_2S_2 & 0 \\ S_2 & C_2S_2 & S_2^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} C_1^2 & C_1S_1 \\ C_1S_1 & S_1^2 \end{bmatrix}$

Ideal homogeneous nonlinear polarizer

Right circular 	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$
Left circular 	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Right elliptical  $\theta = 0°$ $b/a = 0.5$	$\frac{1}{2}\begin{bmatrix} 1 & 0.6 & 0 & 0.8 \\ 0.6 & 0.36 & 0 & 0.48 \\ 0 & 0 & 0 & 0 \\ 0.8 & 0.48 & 0 & 0.64 \end{bmatrix}$	$\frac{2}{5}\begin{bmatrix} 2 & -i \\ i & 0.5 \end{bmatrix}$
Right elliptical  $\theta = 22.5°$ $b/a = 0.318$	$\frac{1}{2}\begin{bmatrix} 1 & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \sqrt{\frac{1}{3}} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \sqrt{\frac{1}{3}} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$	$0.288\begin{bmatrix} 2.73 & 1-i \\ 1+i & 0.733 \end{bmatrix}$
Elliptical, general case	$\frac{1}{2}\begin{bmatrix} 1 & C_2Y & S_2Y & Z \\ C_2Y & C_2^2Y^2 & C_2S_2Y^2 & C_2YZ \\ S_2Y & C_2S_2Y^2 & S_2^2Y^2 & S_2YZ \\ Z & C_2YZ & S_2YZ & Z^2 \end{bmatrix}$	$\begin{bmatrix} \|m\|^2 & mn^* \\ m^*n & \|n\|^2 \end{bmatrix}$ where the Jones major and minor eigenvectors are $\begin{bmatrix} m \\ n \end{bmatrix}$ and $\begin{bmatrix} -n^* \\ m^* \end{bmatrix}$

Other polarizers

Homogeneous, nonscattering, nondepolarizing, nonbirefringent polarizer with principal transmittances $k_1 = p_1^2$ and $k_2 = p_2^2$ and with the transmission axis horizontal ($\theta = 0°$)

$$\frac{1}{2}\begin{bmatrix} k_1+k_2 & k_1-k_2 & 0 & 0 \\ k_1-k_2 & k_1+k_2 & 0 & 0 \\ 0 & 0 & 2\sqrt{k_1k_2} & 0 \\ 0 & 0 & 0 & 2\sqrt{k_1k_2} \end{bmatrix} \qquad \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix}$$

Same but with $\theta = 90°$

$$\frac{1}{2}\begin{bmatrix} k_1+k_2 & -k_1+k_2 & 0 & 0 \\ -k_1+k_2 & k_1+k_2 & 0 & 0 \\ 0 & 0 & 2\sqrt{k_1k_2} & 0 \\ 0 & 0 & 0 & 2\sqrt{k_1k_2} \end{bmatrix} \qquad \begin{bmatrix} p_2 & 0 \\ 0 & p_1 \end{bmatrix}$$

Inhomogeneous right-circular polarizer consisting of two ideal homogeneous layers: linear polarizer with horizontal axis and linear retarder with retardance of 90° and fast axis at 45°; light assumed to be incident on linearly polarizing layer

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ i & 0 \end{bmatrix}$$

Ideal homogeneous linear retarder with retardance $\delta = 90°$

Azimuth ρ of fast axis

0°

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{bmatrix}$$

90°

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

45°

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

$-45°$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix}$$

General

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2^2 & C_2S_2 & -S_2 \\ 0 & C_2S_2 & S_2^2 & C_2 \\ 0 & S_2 & -C_2 & 0 \end{bmatrix} \qquad \begin{bmatrix} C_1^2P + S_1^2Q & \sqrt{2}\, iC_1S_1 \\ \sqrt{2}\, iC_1S_1 & C_1^2Q + S_1^2P \end{bmatrix}$$

Ideal homogeneous linear retarder with retardance $\delta = 180°$

Azimuth ρ of fast axis

$0°$ or $90°$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\pm 45°$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

General

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_2^2 - S_2^2 & 2C_2S_2 & 0 \\ 0 & 2C_2S_2 & S_2^2 - C_2^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad \begin{bmatrix} C_2 & S_2 \\ S_2 & -C_2 \end{bmatrix}$$

Other ideal homogeneous linear retarders

$\delta = 360°$, any ρ

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Any δ, $\rho = 0°$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & D^2 + G^2 & 0 & 0 \\ 0 & 0 & -D^2 + G^2 & 2DG \\ 0 & 0 & -2DG & 2G^2 - 1 \end{bmatrix} \qquad \begin{bmatrix} e^{i\delta/2} & 0 \\ 0 & e^{-i\delta/2} \end{bmatrix}$$

Any δ, any ρ

Mueller matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & D^2 - E^2 + G^2 & 2DE & -2EG \\ 0 & 2DE & -D^2 + E^2 + G^2 & 2DG \\ 0 & 2EG & -2DG & 2G^2 - 1 \end{bmatrix}$$

Jones matrix

$$\begin{bmatrix} C_1^2e^{i\delta/2}+S_1^2e^{-i\delta/2} & C_1S_12i\sin\tfrac{1}{2}\delta \\ C_1S_12i\sin\tfrac{1}{2}\delta & C_1^2e^{-i\delta/2}+S_1^2e^{i\delta/2} \end{bmatrix}$$

or

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

where

$$m_{11} = \cos^2\rho + e^{-i\delta}\sin^2\rho,$$
$$m_{22} = \sin^2\rho + e^{-i\delta}\cos^2\rho,$$
$$m_{12} = m_{21} = (1 - e^{-i\delta})\cos\rho\sin\rho$$

Ideal homogeneous nonlinear retarders

Right circular, $\delta = 90°$ (hence $\zeta = 45°$)

$$\begin{bmatrix} 1&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1&1\\-1&1 \end{bmatrix}$$

Left circular, $\delta = 90°$ (hence $\zeta = -45°$)

$$\begin{bmatrix} 1&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&1 \end{bmatrix} \qquad \frac{1}{\sqrt{2}}\begin{bmatrix} 1&-1\\1&1 \end{bmatrix}$$

Right or left circular, $\delta = 180°$ (hence $\zeta = \pm 90°$)

$$\begin{bmatrix} 1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1 \end{bmatrix} \qquad \begin{bmatrix} 0&1\\-1&0 \end{bmatrix}$$

Right circular, any δ

$$\begin{bmatrix} 1&0&0&0\\0&\cos\delta&\sin\delta&0\\0&-\sin\delta&\cos\delta&0\\0&0&0&1 \end{bmatrix} \qquad \begin{bmatrix} \cos\tfrac{1}{2}\delta & \sin\tfrac{1}{2}\delta\\ -\sin\tfrac{1}{2}\delta & \cos\tfrac{1}{2}\delta \end{bmatrix}$$

Left circular, any δ

$$\begin{bmatrix} 1&0&0&0\\0&\cos\delta&-\sin\delta&0\\0&\sin\delta&\cos\delta&0\\0&0&0&1 \end{bmatrix} \qquad \begin{bmatrix} \cos\tfrac{1}{2}\delta & -\sin\tfrac{1}{2}\delta\\ \sin\tfrac{1}{2}\delta & \cos\tfrac{1}{2}\delta \end{bmatrix}$$

Elliptical, any δ, any ρ

Mueller matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & D^2-E^2-F^2+G^2 & 2(DE+FG) & -2(DF+EG) \\ 0 & 2(DE-FG) & -D^2+E^2-F^2+G^2 & 2(DG-EF) \\ 0 & -2(DF-EG) & -2(DG+EF) & -D^2-E^2+F^2+G^2 \end{bmatrix}$$

Jones matrix

$$\begin{bmatrix} C_r^2 e^{i\delta/2} + S_r^2 e^{-i\delta/2} & C_r S_r(2i \sin \frac{1}{2}\delta) e^{-i\gamma} \\ C_r S_r(2i \sin \frac{1}{2}\delta) e^{i\gamma} & C_r^2 e^{-i\delta/2} + S_r^2 e^{i\delta/2} \end{bmatrix}$$

where $C_r = \cos R$ and $S_r = \sin R$, the quantities R and γ having the significance indicated in Chapters 1 and 2.

BIBLIOGRAPHY

A-1 Abraham, M., and R. Becker, *Classical theory of electricity and magnetism* (Blackie, London, 1937).

A-2 Alexander, N., *Photoelasticity* (Rhode Island State College, Kingston, 1936).

A-3 Allen, P. J., "An instantaneous microwave polarimeter," *Proc. IRE 47*, 1231 (1959).

A-4 Ambronn, H., "Über den Dichroismus pflanzlicher Zellmembranen," *Ann. Physik 34*, 344 (1888).

A-5 Ambronn, H., and A. Frey, *Das Polarisationsmikroskop, seine Verwendung in der Kolloidforschung und in der Farberei* (Akademische Verlag, Leipzig, 1926).

A-6 Ambrose, E. J., A. Elliott, and R. B. Temple, "Use of polarized infrared radiation in the study of crystal structures," *Proc. Roy. Soc. (London) A 206*, 192 (1951).

A-7 *American Institute of Physics handbook* (McGraw-Hill, New York, 1957). Presents data on sheet polarizers, crystals, etc.

A-8 Anderson, S., "Orientation of methylene blue molecules adsorbed on solids," *J. Opt. Soc. Amer. 39*, 49 (1949).

A-9 Angenetter, H., and H. Verleger, "Polarization of Light Emitted from Canal Rays," *Physik. Z. 39*, 328 (1938).

A-10 Anonymous historical account of discoveries on double refraction and polarization, *Edinburgh Phil. J. 1*, 289 (1819).

A-11 Anon., *Lectures on polarized light*, lectures delivered before the Pharmaceutical Society of Great Britain and in the Medical School of the London Hospital (Longman, Brown, Green, and Longman, London, 1843) 110 pp.

A-12 Anonymous article on National Bureau of Standards NBS Sky Compass, *Rev. Sci. Instr. 20*, 460 (1949).

A-13 Anonymous, "Polaroid's vectograph: the system that puts 3-D on one film," *Ideal Kinema 20*, 14 (1954).

A-14 Anonymous article on use of polarizers in three-dimension television system worked out by Pye, Ltd. *British Kinematography 26*, 46 (1955).

A-15 Anonymous article on use of polarizers in 3-D TV system used at Harwell, England, *Nucleonics 15*, 117 (July 1957).

A-16 Anonymous, "Polarization of light of moon and planets," *Sky and Telescope 19*, 17 (1959).
A-17 Appel, A. V., and D. A. Pontarelli, "Infrared polariscope for photoelastic measurements of semiconductors," talk given at meeting of American Optical Society, March 29, 1958; employed Glan-Thompson polarizers, quartz retarders.
A-18 Arago, F. J., *Oeuvres completes*, vol. 10, p. 36 (1812).
— Archambault, C., U.S. patent 2,813,459. Variable-density sunglasses.
A-19 Archard, J. F., and A. M. Taylor, "Improved Glan-Foucault prism," *J. Sci. Instr. 25*, 407 (1948).
A-20 Archard, J. F., "Performance and testing of polarizing prisms," *J. Sci. Instr. 26*, 188 (1949).
A-21 Archard, J. F., P. L. Clegg, and A. M. Taylor, "Photoelectric analysis of elliptically polarized light," *Proc. Phys. Soc. (London) B, 65*, 758 (1952).
A-22 Automobile Manufacturers Association. *Public side of the problem of polarized automobile lights*, pamphlet issued in May 1952, 12 pp. Lists drawbacks to polarized headlights.
A-23 Autrum, H., and H. Stumpf, "Das Bienenauge als Analysator für Polarisiertes Licht," *Z. Naturforsch., Pt. b, 5*, 116 (1950).
— Baerwald, H. G., U.S. patent 2,766,659 (1956). An electro-optic modulator of great complexity.
— Bailey, E. D., and M. M. Brubaker, U.S. patent 2,246,087 (1941). Describes methods of stretching various materials as for improving the performance of the resulting polarizer.
B-1 Baird Associates, Technical Circular RD 505 (1951). Describes a modified Lyot-Öhman filter priced at $3000.
B-2 Barer, R., "Variable colour-amplitude phase-contrast microscopy," *Nature 164*, 1087 (1949).
B-3 Barer, R., "Some experiments with polarizing films in the ultraviolet," *J. Sci. Instr. and Phys. in Ind. 26*, 325 (1949). Describes use of a PVA-iodine polarizer in the near ultraviolet.
B-4 Barer, R., "Phase contrast and interference microscopy," in D. A. Spencer *Progress in photography* (Focal Press, London), Vol. 2, 1951–1954.
B-5 Bartholinus, E., *Experimenta crystalli Islandici disdiaclastici quibus mira & insolita refractio detegetur* (Hafnia [Copenhagen], 1670). Announces discovery of double refraction.
B-6 Bates, F. J., *Polarimetry, saccharimetry and the sugars*, National Bureau of Standards. Circular C-440 (1942).
B-7 Baxter, L., private communication.
B-8 Baxter, L., A. S. Makas, and W. A. Shurcliff, "Measuring spectral properties of high-extinction polarizers," *J. Opt. Soc. Amer. 46*, 229 (1956). Describes rotating holder, and H and K polarizers for which G_{90} exceeds 5.0.
B-9 Baxter, L., "On the properties of polarization elements as used in optical instruments. III. Angular aperture functions of a positive dichroic film polarizer," *J. Opt. Soc. Amer. 46*, 435 (1956). Discusses obliquity effects in HN-22.
B-10 Bayley, H. G., "Gelatin as a photo-elastic material," *Nature 183*, 1757 (1959).
B-11 Baylor, E. R., and F. E. Smith, "Orientation of *Cladocera* to polarized light," *Am. Naturalist 87*, 97 (1953).

B-12 Bennett, H. E., J. M. Bennett, and M. R. Nagel, "Question of the polarization of infrared radiation from the clear sky," *J. Opt. Soc. Amer. 51*, 237, (1961).

B-13 Bennett, H. S., "Microscopical investigation of biological materials with polarized light," in Ruth McClung, ed., *McClung's handbook of microscopical technique* (Hoeber, New York, 1949).

B-14 Berkman, S., J. Boehm, and H. Zocher, "Anisotropes Kupfer, Silber, und Gold," *Z. Physik. Chem. 124*, 83 (1926), article on dichroism of aligned microcrystals of gold, silver, mercury, etc.

B-15 Bernauer, F., "Neue Wege zur Herstellung von Polarisatoren," *Fortschr. Mineral. Krist. Petrog. 19*, 22 (1935).

B-16 Bertrand, E., "Optique — sur un nouveau prisme polarisateur," *Compt. rend. 96*, 538 (1884).

B-17 Besse, A., and F. Desvignes, "An infrared polariscope," *Rev. opt. 38*, 344 (1959). Device employing silicon mirrors at Brewster's angle.

B-18 Beth, R. A., "Mechanical detection and measurement of the angular momentum of light," *Phys. Rev. 50*, 115 (1936). Deals with torque produced by circularly polarized light.

B-19 Bhagavantam, S., "Polarization of Raman lines in liquids," *Indian J. Phys. 7*, 79 (1932).

B-20 Billardon, M., and J. Badoz, "Spectropolarimètre photoélectrique destiné à l'étude de la dispersion du pouvoir rotatoire naturel," *Compt. rend. 248*, 2466 (1959).

B-21 Billings, B. H., "Tunable narrow-band optical filter," *J. Opt. Soc. Amer. 37*, 738 (1947).

B-22 Billings, B. H., and E. H. Land, "Comparative survey of some possible systems of polarized headlights," *J. Opt. Soc. Amer. 38*, 819 (1948).

B-23 Billings, B. H., "Monochromatic depolarizer," *J. Opt. Soc. Amer. 41*, 966 (1951).

B-24 Billings, B. H., "Un depolarizador monocromatico," *Ciencia e invest. (Buenos Aires) 8*, 99 (1952).

B-25 Biot, J. R., "Sur un mode particulier de polarisation qui s'observe dans la tourmaline, "*Bull. soc. philomath. Paris 6*, 26 (1815).

B-26 Bird, G. R., M. Parrish, Jr., and E. R. Blout, "Apparatus for the observation of infrared streaming dichroism of polymer solutions," *Rev. Sci. Instr. 29*, 305 (1958).

B-27 Bird, G. R., and M. Parrish, Jr., "The wire grid as a near-infrared polarizer," *J. Opt. Soc. Amer. 50*, 886 (1960).

B-28 Bird, G. R., and W. A. Shurcliff, "Pile-of-plates polarizers for the infrared: improvement in analysis and design," *J. Opt. Soc. Amer. 49*, 235 (1959).

B-29 Birge, R. T., and L. A. DuBridge, "Nature of unpolarized light," *J. Opt. Soc. Amer. 25*, 179 (1935).

B-30 Blackwell, H. R., "Effect of tinted optical media upon visual efficiency at low luminance," *J. Opt. Soc. Amer. 43*, 815 (1953). Points out harm of tinted windshields and tinted glasses.

B-31 Blake, R. P., A. S. Makas, and C. D. West, "Molecular-type dichroic film polarizers for 0.75 to 2.8 micron radiations," *J. Opt. Soc. Amer. 39*, 1054 (1949). Describes HR polarizer.

— Blake, R. P., U.S. patent 2,494,686. Infrared polarizer.

B-32 Blout, E. R., and R. Karplus, "Infrared spectrum of polyvinyl alcohol," *J. Am. Chem. Soc. 70*, 862 (1948).

B-33 Blout, E. R., G. R. Bird, and D. S. Grey, "Infrared microspectroscopy," *J. Opt. Soc. Amer. 39*, 1052A (1949).

B-34 Blout, E. R., G. R. Bird, and D. S. Grey, "Infrared microspectroscopy," *J. Opt. Soc. Amer. 40*, 304 (1950).

B-35 Blout, E. R., and G. R. Bird, "Infrared microspectroscopy, II," *J. Opt. Soc. Amer. 41*, 547 (1951).

B-36 Blout, E. R., talk on "Chemical optics" to New England Section of the Optical Society of America, May 20, 1954.

B-37 Boehm, G., and R. Signer, "Über die Strömungsdoppelbrechung von Eiweisslösungen," *Helv. Chim. Acta. 14*, 1370 (1931). Article on flow birefringence.

B-38 Boehm, G., "Über maculare (Haidinger'sche) Polarisationbüschel und über einen polarisationoptischen Fehler des Auges," *Acta Ophthalmol. 18*, 109 (1940).

B-39 Bolla, G. V., "Polarisationseffekte in Quarzspektrographen," *Z. Physik 103*, 756 (1936).

B-40 Bond, W. L., and J. Andrus, "Photograph of the stress field around edge dislocations," *Phys. Rev. 101*, 1211 (1956).

— Bond, W. L., U.S. patent 2,768,557. Electro-optic shutter compensated for divergent rays.

B-41 Born, M., *Optik* (Springer, Berlin, 1933).

B-42 Born, M., "Theory of optical activity," *Proc. Roy. Soc. (London) A 150*, 84 (1935).

B-43 Born, M., and E. Wolf, *Principles of optics* (Pergamon, New York, 1959).

B-44 Bouasse, H., *Optique cristalline double refraction, polarisation rectiligne et elliptique* (Librairie Delegrave, Paris, 1925).

B-45 Bouhet, C., and Lafont, "New normal field polarizers," *Rev. opt. 28*, 490 (1949). Describes a square-ended prism-type polarizer.

B-46 Bouriau, Y., and J. Lenoble, "Etudé des polariseurs pour l'ultraviolet," *Rev. opt. 36*, 531 (1957). Describes various cemented-prism-type, ultraviolet polarizers.

B-47 Bovis, P., "Spectres d'absorption et pléochroïsme de l'iode et de l'hérapathite," *Compt. rend. 184*, 1237 (1927). States the dichroic ratio of individual crystals of iodine.

B-48 Brewster, D., "On the laws which regulate the polarisation of light by reflexion from transparent bodies," paper read on March 16, 1815, published in *Phil. Trans. 105*, 125 (1815).

— Bridges, S. W., and J. R. Roehrig, U.S. patent 2,824,488. Polarizing-microscope apparatus for grading fibers.

B-49 Brode, W. R., and C. H. Jones, "Recording spectrophotometer and spectropolarimeter," *J. Opt. Soc. Amer. 31*, 743 (1941).

B-50 Brode, W. R., "Optical rotation of polarized light by chemical compounds," *J. Opt. Soc. Amer. 41*, 987 (1951).

— Brown, C. H., U.S. patents 2,224,214 and 2,287,598. Polarizer employing wires.

B-51 Brown, T. B., "Elliptic polarimeter for the student laboratory: specimens of elliptically polarized light," *Am. J. Phys. 26*, 183 (1958).

B-52 Browne, C. A., and F. W. Zerban, *Physical and chemical methods of sugar analysis* (Wiley, New York, 1941).

B-53 Bruhat, G., "Le dichroïsme circulaire," *Rev. opt. 8*, 365, 413 (1929).

B-54 Bruhat, G., *Traité de polarimetrie* (Editions Revue d'Optique, Paris, 1930).

B-55 Bruhat, G., and P. Guenard, "Étude du dichroïsme circulaire de solutions de camphre dans des solvants organiques," *Compt. rend. 203*, 784 (1936).

B-56 Buijs, K., "Preparation of selenium polarizers for the near infrared region," *Appl. Spectroscopy 14*, 81 (1960).

B-57 Burri, C., *Das Polarisationsmikroskop* (Birkhauser, Basel, 1950).

C-1 Cabannes, J., and P. Daure, "Sur le spectre Raman du benzène en lumière circulaire," *Compt. rend. 208*, 1700 (1939).

C-2 Campbell, C. D., "Sky compass for field use," *Northwest Sci. 28*, 43 (1954).

C-3 Carpenter, R. O., "Electro-optic effect in uniaxial crystals of the dihydrogen phosphate type, III. Measurements of coefficients," *J. Opt. Soc. Amer. 40*, 225 (1950). Discusses smallest measurable changes.

C-4 Cayrel, R., and L. Schatzman, "Nouveaux polariseurs et leur procédé de fabrication," French patent 1,109,170 (July 1954).

C-5 Central Scientific Co., *Cenco News Chats* (No. 86, 1959). Discussion of equipment for demonstrating polarization of microwaves.

C-6 Cerf, R., and H. A. Scheraga, "Flow birefringence in solutions of macromolecules," *Chem. Revs. 51*, 185 (1952).

C-7 Chandrasekhar, S., "On the radiative equilibrium of a stellar atmosphere. X," *Astrophys. J.*, *103*, 350 (1946). Prediction on polarization of starlight.

C-8 Chandrasekhar, S., *Radiative transfer* (Clarendon Press, Oxford, 1950), 393 pp.

C-9 Chandrasekhar, S., "The optical rotatory power of quartz and its variation with temperature," *Proc. Indian Acad. Sci. 35*, 103 (1952).

C-10 Chandrasekhar, S., and D. D. Elbert, "Illumination and polarization of the sunlit sky on Rayleigh scattering," *Trans. Am. Phil. Soc. 44*, Pt. 6, 88 (1954).

C-11 Chandrasekaran, K. S., "Perfect polarization of x-rays by crystal reflection," *Proc. Indian Acad. Sci. A 44*, 387 (1956).

C-12 Chapman, J. A., "Device for visualizing the pattern of plane polarized light from blue sky," *Nature 181*, 1393 (1958).

C-13 Charney, E., "Dichroic ratio measurements in the infrared region," *J. Opt. Soc. Amer. 45*, 980 (1955). Discusses errors due to polarization by the spectrophotometer prism.

C-14 Charru, M. A., "Expériences à 3000 MHz sur les aériens hélicoïdaux; réalisation et analyse d'un dichroïsme circulaire," *J. phys. radium 21*, 93 S (1960). Describes experiments in which right and left helical antennas were employed.

C-15 Choudhuri, K., "On the polarized fluorescence-dyestuffs in solution," *Indian J. Phys. 18*, 74 (1944).

— Chubb, L. W., U.S. patent 2,087,795. Headlight system employing polarizers at 45°.

C-16 Chubb, L. W., Jr., D. S. Grey, E. R. Blout, and E. H. Land, "Properties of polarizers for filters and viewers for 3-D motion pictures," *J. Soc. Motion Picture Television Engrs. 62*, 120 (1954).

C-17 Clarke, J. T., and E. R. Blout, "Nature of the carbonyl groups in polyvinyl alcohol," *J. Polymer Sci. 1*, 419 (1946).

C-18 Cohen, M. H., "Radio astronomy polarization measurements," *Proc. I.R.E. 46*, 172 (1958).

C-19 Cohen, S. G., H. C. Haas, and H. Slotnick, "Studies on hydroxyethylpolyvinyl alcohol," *J. Polymer Sci. 11*, 193 (1953). Shows x-ray diffraction pattern for PVA and related materials.

C-20 Coker, E. G., and L. N. G. Filon, *A treatise on photo-elasticity* (Cambridge University Press, New York, ed. 2, rev. by H. T. Jessop, 1957), 720 pp.

C-21 Colman, K. W., D. Courtney, J. B. Freeman, and R. Bernstein, "The control of specular reflections from bright tube radar displays," Nov. 15, 1958, Report No. 23 by Courtney & Co., Philadelphia, for Engineering Psychology Branch of Office of Naval Research, contract Nonr-2346(00).

C-22 Compton, W. D., "Polarized light as a tool for studying the symmetry properties of color centers," talk at March 1958 meeting of American Physical Society.

C-23 Condon, E. U., "Theories of optical rotatory power," *Revs. Mod. Phys. 9*, 432 (1937).

C-24 Conn, G. K. T., and F. J. Bradshaw, *Polarized light in metallography* (Butterworth, London, 1952).

C-25 Conn, G. K. T., and G. D. Eaton, "On the use of a rotating polarizer to measure optical constants in the infrared," *J. Opt. Soc. Amer. 44*, 484 (1954). Claim 99.7-percent polarizance for an eight-film selenium pile of plates.

C-26 Conn, G. K. T., and G. D. Eaton, "On polarization by transmission with particular reference to selenium films in the infrared," *J. Opt. Soc. Amer. 44*, 553 (1954).

C-27 Conroy, J., "Polarization of light by crystals of iodine," *Proc. Roy. Soc. (London) 25*, 51 (1876).

C-28 Cords, O., "Das Haidingersche Büschel und seine Erklärung als Beitrag für eine physikalische Deutung des Sehvorganges," *Optik 2*, 423 (1947).

C-29 Cotton, A., "Absorption inégale des rayons circulaires droit et gauches dans certains corps actifs," *Compt. rend. 120*, 989 (1895). Discovery of circular dichroism in solutions.

C-30 Cotton, A., "Prismes polariseurs a champ normal fondés sur la reflexion crystalline interne," *Compt. rend. 193*, 268 (1931). Describes the Cotton polarizer.

C-31 Coulson, K. L., D. Deirmendjian, R. S. Fraser, C. Seaman, and Z. Sekera, *Investigation of polarization of skylight* (University of California, Department of Meteorology, June 1955), 198 pp.

C-32 Culver, W. H., "The maser," *Science 126*, 810 (1957).

D-1 D'Agostino, J., D. C. Drucker, C. K. Liu, and C. Mylonas, "Analysis of plastic behavior of metals with bonded birefringent plastics," *Proc. Soc. Exp. Stress Anal. 12*, 115 (1955).

D-2 D'Agostino, J., D. C. Drucker, C. K. Liu, and C. Mylonas, "Epoxy adhesives and casting resins as photoelastic plastics," *Proc. Soc. Exp. Stress Anal. 12*, 123 (1955).

D-3 Dawson, E. F., and N. O. Young, "Helical Kerr cell," *J. Opt. Soc. Amer. 50*, 170 (1960). Confirms calculations easily made with the aid of the Jones calculus that a skewed series of linear retarders is equivalent to a single circular retarder.

D-4 Dehmelt, H. G., "Slow spin relaxation of optically polarized sodium

atoms," *Phys. Rev. 105*, 1487 (1957). Deals with absorption of circularly polarized light in masers.

D-5 Demon, L., "Propriétés optiques des cristaux de bleu de méthylène," *Ann. phys. 1*, 101 (1946). Describes rubbing glass surfaces and applying dyes.

D-6 de Vaucouleurs, G., *Physics of the planet Mars* (Macmillan, New York, 1954). Reviews use of polarizers in determining the amount of atmosphere on Mars.

D-7 Dévé, C., *Optical workshop principles* (Hilger and Watts, London, 1954). Discusses manufacture of polarizing prisms.

D-8 de Vries, H., R. Jielof, and A. Spoor, "Properties of the human eye with respect to linearly and circularly polarized light," *Nature 166*, 958 (1950).

D-9 Dietze, G., *Einfuhrung in die Optik der Atmosphäre* (Akademisch Verlag, Leipzig, 1957), 263 pp. Discusses polarization.

D-10 Ditchburn, R. W., *Light* (*Interscience*, New York, 1953), 680 pp.

D-11 Ditchburn, R. W., "Some new formulas for determining the optical constants from measurements on reflected light," *J. Opt. Soc. Amer. 45*, 743 (1955).

D-12 Dmitri, I., "Two new tools for color," *Photography 9*, 68 (October 1954). Describes 1954 General Electric variable color filters.

D-13 Dobrowolski, J. A., "Mica interference filters with transmission bands of very narrow half-width," *J. Opt. Soc. Amer. 49*, 794 (1959).

D-14 Dollfus, A., "Détermination de la pression atmosphérique sur la planète Mars," *Compt. rend. 232*, 1066 (1951).

D-15 Downie, A. R., "Removal of signal fluctuations in a photoelectric polarimeter," *J. Sci. Instr. 35*, 114 (1958).

D-16 Dreyer, J. F., "A variable light polarizing coating," *J. Opt. Soc. Amer. 37*, 983 (1947). Discusses Beilby layers.

D-17 Dreyer, J. F., and C. W. Ertel, "Orientation of the surface of glass," *Glass Ind. 29*, 197 (1948).

D-18 Dreyer, J. F., "Some of the applications of polarized light to photography," *PSA Journal 20*, 28 (June 1954).

— Dreyer, J. F., U.S. patents:

2,400,877 (1946). Dichroic polarizer.

2,432,867 (1947). Light-polarizing coating on curved surface such as that of a lamp.

2,481,830 (1949). Method of preparing a dichroic polarizer; the method employs an intermediate dichroic film.

2,484,818 (1949). Polarizing mirror involving nematic state.

2,544,659 (1951). Dichroic polarizer.

2,776,598 (1957). Polarizing mirror involving nematic state.

D-19 Dreyfus, M., "Measurement of birefringence," *J. Opt. Soc. Amer. 46*, 142 (1956).

D-20 du Bois, H., and H. Rubens, "Polarisation ungebeugter langwelliger Wärmstrahlen durch Drahtgitter," *Ann. Physik 35*, 243 (1911).

D-21 Dudley, B., "Vectograph stereograms," *Photo Tech. 3*, 30 (May 1941).

D-22 Dufay, J., *Galactic nebulae and interstellar matter* (Philosophical Library, New York, about 1957). Chapter 15 discusses polarization of the light from stars by interstellar matter.

D-23 Dunsmuir, P., "Use of polarized light for the examination of etched metal crystals and their orientation," *Brit. J. Appl. Phys. 3*, 264 (1952).

D-24 DuPont (E. I. duPont de Nemours and Company), *DuPont ELVANOL polyvinyl alcohols* (DuPont, Wilmington, Delaware, 1953).

D-25 Duverney, R., and A. M. Vergnoux, "Polarimétrie dans l'infra-rouge," *J. phys. radium 18*, 527 (1957). A long review article comparing many types of infrared polarizers.

D-26 Dyson, J., "Interferometer for straightness measurement," *Nature 175*, 559 (1955). Describes use of polarizers and concave mirrors in measuring straightness as a probe (Wollaston prism) is moved along the alleged straight line.

E-1 Eastman Kodak Co., *Photography by polarized light* (Pamphlet 4-36-CH-10; April 1936), 32 pp.

E-2 Eastman Kodak Co., *Photography by polarized light* (booklet, 1938), 40 pp.

E-3 Eastman Kodak Co., *Eastman Pola-Screens* (Pamphlet 8-38-CH-25, 1938), 6 pp.

E-4 Eastman Kodak Co., *Kodak filters and Pola-Screens for black-and-white photography* (booklet, ed. 4, 1956), 54 pp.

E-5 Edgerton, H. E., and C. W. Wyckoff, "A rapid-action shutter with no mechanical moving part," *J. Soc. Motion Picture Engrs. 56*, 398 (1951).

E-6 Edsall, J. T., A. Rich, and M. Goldstein, "An instrument for the study of double refraction of flow at low and intermediate velocity gradients," *Rev. Sci. Instr. 23*, 695 (1952).

E-7 Edwards, D. F., and M. J. Bruemmer, "Polarization of infrared radiation by reflection from germanium surfaces," *J. Opt. Soc. Amer. 49*, 860 (1959).

E-8 Elliott, A., and E. J. Ambrose, "Polarization of infra-red radiation," *Nature 159*, 641 (1947). Discusses a transmission-type polarizer using three selenium films.

E-9 Elliott, A., E. J. Ambrose, and R. B. Temple, "Polarization of infra-red radiation," *J. Opt. Soc. Amer. 38*, 212 (1948). Describes a device employing selenium.

E-10 Elliott, A., E. J. Ambrose, and R. B. Temple, "Double orientation and infra-red dichroism in polymers," *Nature 163*, 567 (1949). Discusses polyvinyl alcohol, nylon, etc.

E-11 Elliott, A., "Infra-red dichroism and chain orientation in crystalline ribonuclease," *Proc. Roy. Soc. (London) A 211*, 490 (1952).

E-12 Elliott, A., "Infra-red dichroism in synthetic polypeptides," *Nature 172*, 359 (1953).

E-13 Ellis, J. W., and J. Bath, "The near infra-red absorption spectrum of sucrose crystals in polarized light," *J. Chem. Phys. 6*, 221 (1938). Discusses polarizing prisms suitable for use in near infrared.

E-14 Ellis, J. W., and L. Glatt, "Channeled infra-red spectra produced by birefringent crystals," *J. Opt. Soc. Amer. 40*, 141 (1950). Discusses inadvertent polarization by prisms in infrared spectrophotometers.

E-15 Eshbach, J. R., and M. W. P. Strandberg, "Apparatus for Zeeman effect measurement of microwave spectra," *Rev. Sci. Instr. 23*, 623 (1952).

E-16 Evans, J. W., "The birefringent filter," *J. Opt. Soc. Amer. 39*, 229 (1949). Discusses the Lyot-Öhman filter.

E-17 Evans, J. W., "Šolc birefringent filter," *J. Opt. Soc. Amer. 48*, 142 (1958).

F-1 Fagg, L. W., and S. S. Hanna, "Polarization measurements on nuclear gamma rays," *Revs. Mod. Phys. 31*, 711 (1959). Comprehensive article on the production and detection of polarized gamma rays.

F-2 Fahy, E. F., and M. A. MacConaill, "Optical properties of cellophane," *Nature 178*, 1072 (1956).

F-3 Falkoff, D. L., and J. E. McDonald, "On the Stokes parameters for polarized radiation," *J. Opt. Soc. Amer. 41*, 861 (1951).

F-4 Fallon, J., unpublished manuscript written in about 1950.

F-5 Fano, U., "Remarks on the classical and quantum-mechanical treatment of partial polarization," *J. Opt. Soc. Amer. 39*, 859 (1949). Relates Stokes parameters to quantum-mechanical properties of photons.

F-6 Fano, U., "Description of states in quantum mechanics by density matrix and operator techniques," *Revs. Mod. Phys. 29*, 74 (1957).

F-7 Farwell, H. W., "Scattered light from Polaroid plates," *J. Opt. Soc. Amer. 28*, 460 (1938).

F-8 Federov, F. I., "On the theory of optical activity in crystals," *Optics and Spectroscopy 6*, 49 (1959).

F-9 Filon, L. N. G., *Manual of photo-elasticity for engineers* (Cambridge University Press, Cambridge, 1936), 140 pp.

F-10 Finch, D. M., J. M. Chorlton, and H. F. Davidson, "The effect of specular reflection on visibility," in *Proceedings of the Fourteenth Session of the International Commission on Illumination* (Illuminating Engineering Society, New York, 1959).

F-11 Foitzik, L., and K. Lenz, "Einfluss des Aerosols auf die Himmelslichtpolarisation," *Optik 17*, 554 (1960).

F-12 Försterling, K., *Lehrbuch der Optik* (Hirzel, Leipzig, 1928). Contains long section on crystals, birefringence, etc.

F-13 Fox, A. G., S. E. Miller, and M. T. Weiss, "Behavior and applications of ferrites in the microwave region," *Bell System Tech. J. 34*, 5 (1955).

F-14 Françon, M., "Polarization apparatus for interference microscopy and macroscopy of isotropic transparent objects," *J. Opt. Soc. Amer. 47*, 528 (1957).

F-15 Fresnel, A. J., "Sur la diffraction de la lumière, où l'on examine particulièrement le phénomène des franges colorées que présentent les ombres des corps éclairés par un point lumineux," *Ann. chim. (Paris)* [2] *1*, 239 (1816). Describes noninterference of orthogonally polarized rays.

F-16 Friedman, G. H., "An elliptical polarization synthesizer," Communication and Electronics, July, 1955.

F-17 Frisch, K. von, *Bees: their vision, chemical sense, and language* (Cornell University Press, Ithaca, 1950).

F-18 Frocht, M. M., *Photoelasticity* (Wiley, New York, 1941).

F-19 Frocht, M. M., and R. Guernsey, "Studies in three-dimensional photoelasticity," *Proceedings of the First U.S. National Congress of Applied Mechanics* (American Society of Mechanical Engineers, New York, 1951), p. 301.

— Frost, R. H., U.S. patent 2,856,810. Automobile sun visor.

F-20 Fuessner, K., "Ueber die Prismen zur Polarisation des Lichtes," *Z. Instrumentenk. 4*, 47 (1884).

G-1 Gabler, F., and P. Sokob, "Senarmont compensator," *Z. Instrumentenk. 58*, 301 (1938).

G-2 Gänge, C., *Polarisation des Lichtes* (Quandt & Händel, Leipzig, 1894).

G-3 Ganguly, S. C., and N. K. Chaudhury, "Anisotropy of fluorescence of some organic crystals," *Phys. Rev. 95*, 1148 (1954).

G-4 General Electric Company, *How to use your variable color filter* (Pamphlet GE-J-2411A, 11-54), 16 pp.

G-5 George, W. H., "Production of polarized x-rays," *Proc. Roy. Soc.* (*London*) *A 156*, 96 (1936).

G-6 Gillham, E. J., and R. J. King, "New design of spectropolarimeter," *J. Sci. Instr. 38*, 21 (1961).

G-7 Godina, D. A., and G. P. Faerman, "Colloidal suspensions of herapathite for the construction of polarizing luminous filters," *J. Appl. Chem.* (*U.S.S.R.*) *14*, 362 (1941). Describes making suspensions of herapathite crystals.

G-8 Godina, D. A., "Optical properties of polarizing filters made of polyvinyl alcohol," *J. Tech. Phys.* (*U.S.S.R.*) *18*, 1317 (1948).

G-9 Goetze, R., "La ley de Malus y su comprabación experimental," paper presented in Caracas, Venezuela, at the February 1955 meeting of the Venezuelan Association for the Advancement of Science (AVAC). Presents data on the degree of polarization in light emitted at grazing angles from tungsten filaments.

G-10 Grabau, M., "Optical properties of Polaroid for visible light," *J. Opt. Soc. Amer. 27*, 420 (1937).

G-11 Grabau, M., "Polarized light enters the world of everyday life," *J. Appl. Phys. 9*, 215 (1938).

G-12 Grabau, M., *Introduction to polarized light and its applications* (Pamphlet, Polaroid Corporation, Cambridge, Massachusetts, 1940), 46 pp. (A revised version was prepared by the Polaroid Corporation in 1945; see P-22).

G-13 Groosmuller, J. T., "Das Polarisationsfeld Nicolscher Prismen," *Z. Instrumentenk. 46*, 563 (1926). Excellent discussion of obliquity effects in one nicol or two crossed nicols.

G-14 Gurnee, E. F., "Theory of orientation and double refraction in polymers," *J. Appl. Phys. 25*, 1232 (1954).

H-1 Haas, H. C., "Note on the infrared absorption spectrum of polyvinyl alcohol," *J. Polymer Sci. 26*, 391 (1957).

H-2 Haase, M., "Dichroitische Kristalle und ihre Verwendung für Polarisationsfilter," *Zeiss Nachr. 2*, 55 (August 1936).

H-3 Haase, M., "Beispiele zur Wirkungsweise der Polarisationsfilter," *Zeiss Nachr. 2*, 55 (August 1936).

H-4 Haase, H. H., "Neue Polarizations filter und der Verwendung Dichroitischer Kristalle," *Z. tech. Phys. 18*, 69, 1937. Mentions Zeiss Herotare and Mipolare linearly polarizing filters and discusses obliquity effects in certain types of polarizers.

H-5 Haber, H., "Safety hazard of tinted automobile windshields at night," *J. Opt. Soc. Amer. 45*, 413 (1955). States that use of isotropic dyes in windshields or "night-glasses" is harmful in night driving.

H-6 Haidinger, W., "Über das direkte Erkennen des polarisierten Lichts und der Lage der Polarisationsebene," *Ann. Physik 63*, 29 (1844).

H-7 Haidinger, W., "Ueber den Pleochroismus des Amethysts," *Ann. Physik 70*, 531 (1847). Discovery of circular dichroism.

H-8 Hall, J. S., and A. H. Mikesell, *Polarization of light in the galaxy as determined from observations of 551 early-type stars* (Government Printing Office, Washington, 1950; U.S. Naval Observatory Publications, vol. 17, pt. 1), p. 16.

H-9 Hall, J. S., "Some polarization measurements in astronomy," *J. Opt. Soc. Amer. 41*, 963 (1951).

H-10 Hallimond, A. F., *Manual of the polarizing microscope* (Cooke Troughton and Simms, Ltd., York, England, n. d.).

H-11 Hallimond, A. F., "Use of Polaroid for the microscope," *Nature 154*, 369 (1944).

H-12 Hardy, A. C., "A new recording spectrophotometer," *J. Opt. Soc. Amer. 25*, 305 (1935). Mentions spectropolarimeter.

H-13 Hariharan, P., "Accurate measurements of phase differences with the Babinet compensator," *J. Sci. Instr. 37*, 278 (1960). Device produces fringes by a double-pass method.

H-14 Harrick, N. J., "Reflection of infrared radiation from a germanium-mercury interface," *J. Opt. Soc. Amer. 49*, 376 (1959).

H-15 Harrick, N. J., "Infrared polarizer," *J. Opt. Soc. Amer. 49*, 379 (1959). Employs germanium-mercury interface. Effective from 2 to 200 μ.

H-16 Hartshorne, N. H., and A. Stuart, *Crystals and the polarizing microscope* (Arnold Press, London, ed. 2, 1950).

H-17 Hartwig, G., and H. Schopper, "Circular polarization of internal bremsstrahlung emitted in the K capture of A^{37}," *Bull. Am. Phys. Soc.* [2] *4*, 77 (1959). Abstract.

H-18 Heller, W., "Polarimetry," in A. Weissberger, ed., *Physical methods in organic chemistry*, vol. 1, part 3 (Interscience, New York, ed. 3, 1960), chap. 33.

H-19 Helmholtz, H. von, *Physiological optics*, ed. J. P. C. Southall (Optical Society of America, 1924), 3 vols.

H-20 Helwich, O., *Wissenschaftliche Photographie* (Helwich, Darmstadt, 1958).

H-21 Henroit, E., "Les couples exercés par la lumière polarisée circulairement," *Compt. rend. 198*, 1146 (1934).

H-22 Henry, F. G., "Improved lighting in the Bendix and Gilfillan GCA operations trailers," Technical Memorandum No. TM-293, June 25, 1958, issued by U.S. Navy Electronics Laboratory, San Diego.

H-23 Herapath, W. B., "On the optical properties of a newly-discovered salt of quinine which crystalline substance possesses the power of polarizing a ray of light, like tourmaline, and at certain angles of rotation of depolarizing it, like selenite," *Phil. Mag.* [4] *3*, 161 (1852). Reports discovery of the crystal herapathite.

H-24 Herapath, W. B., "Further researches into the properties of the sulphate of iodo-quinone or herapathite," *Phil. Mag. 9*, 366 (1855).

H-25 Hetényi, M., *Handbook of experimental stress analysis* (Wiley, New York, 1950), 1080 pp.

H-26 Hiltner, W. A., "On the presence of polarization in the continuous radiation of early-type stars," *Astrophys. J. 106*, 231 (1947).

H-27 Hiltner, W. A., "Polarization of light from distant stars by interstellar medium," *Science 109*, 165 (1949).

H-28 Hiltner, W. A., "On the presence of polarization in the continuous radiation of stars. II," *Astrophys. J. 109*, 471 (1949).

H-29 Hiltner, W. A., "On polarization of radiation by interstellar medium," *Phys. Rev. 78*, 170 (1950).

H-30 Hiltner, W. A., "Polarization of stellar radiation, III. The polarization of 841 stars," *Astrophys. J. 114*, 241 (1951).

H-30a Hiltner, W. A., "Polarization of the Crab Nebula," *Astrophys. J. 125*, 300 (1957). Attributes the polarization to a synchrotron mechanism.

H-31 Hiltner, W. A., "Photoelectric polarization observations of the jet in M87," *Astrophys. J. 130*, 340 (1959).

H-32 Holbourn, A. H. S., "Angular momentum of circularly polarized light," *Nature 137*, 31 (1936).

H-33 Howard, F. J., J. M. Hood, and S. S. Ballard, "A three-polarizer calibration unit for photometric instruments," *J. Opt. Soc. Amer. 45*, 904 (1955).

H-34 Hsu, H., M. Richartz, and Y. Liang, "A generalized intensity formula for a system of retardation plates," *J. Opt. Soc. Amer. 37*, 99 (1947).

H-35 Hughes, R. H., "Modified Wollaston prism for spectral polarization studies," *Rev. Sci. Instr. 31*, 1156 (1960). Use of thin wedges of quartz as pseudo depolarizers.

H-36 Hulburt, E. O., "Polarization of light at sea," *J. Opt. Soc. Amer. 24*, 35 (1934).

H-37 Hulburt, E. O., "Sextant with improved filters," *J. Opt. Soc. Amer. 26*, 216 (1936).

H-38 Hull, G. F., Jr., "Microwave experiments and their optical analogues," appendix in J. Strong, S-31.

H-39 Hurlbut, C. S., Jr., and J. L. Rosenfeld, "Monochromator utilizing the rotary power of quartz," *Am. Mineralogist 37*, 158 (1952).

— Hurlbut, C. S., Jr., U.S. patent 2,742,818. Monochromator using several quartz plates of unlike thickness and several linear polarizers.

H-40 Hurwitz, H., Jr., and R. C. Jones, "A new calculus for the treatment of optical systems, II. Proof of three general equivalence theorems," *J. Opt. Soc. Amer. 31*, 493 (1941).

H-41 Hurwitz, H., Jr., "The statistical properties of unpolarized light," *J. Opt. Soc. Amer. 35*, 525 (1945).

H-42 Huyghens, C., *Traité de la lumière* (Leyden, 1690). Announces the discovery of polarized light.

H-43 Hyde, W. L., "Polarization techniques in the infrared," *J. Opt. Soc. Amer. 38*, 663 (1948).

H-44 Hyde, W. L., E. F. Tubbs, and C. J. Koester, "An automatic photoelectric polarimeter," *J. Opt. Soc. Amer. 49*, 513 (1959).

I-1 Ingersoll, L. R., and D. H. Liebenberg, "Faraday effect in gases and vapors. I," *J. Opt. Soc. Amer. 44*, 566 (1954).

I-2 Ingersoll, L. R., and D. H. Liebenberg, "Faraday effect in gases and vapors. II," *J. Opt. Soc. Amer. 46*, 538 (1956).

I-3 Ingersoll, L. R., and D. H. Liebenberg, "Faraday effect in gases and vapors. III," *J. Opt. Soc. Amer. 48*, 339 (1958).

I-4 Inoué, S., "Polarization-optical studies of the mitotic spindle," *Chromosoma 5*, 487 (1953). Describes an improved polarizing microscope.

I-5 Inoué, S., and W. L. Hyde, "A device to obtain high extinction at high apertures in polarizing microscopes," *J. Opt. Soc. Amer. 46*, 372 (1956).

I-6 Inoué, S., and W. L. Hyde, "Studies on depolarization of light at microscope lens surfaces. II. The simultaneous realization of high resolution and high sensitivity with the polarizing microscope," *J. Biophys. Biochem. Cytol. 3*, 831 (1957).

I-7 Inoué, S., and C. J. Koester, "Optimum half-shade angle in polarizing instruments," *J. Opt. Soc. Amer. 49*, 556 (1959).

I-8 Insley, H., and V. D. Frechette, *Microscopy of ceramics and cements* (Academic Press, New York, 1955), 286 pp.

I-9 Ivanoff, A., "Degree of polarization of submarine illumination," *J. Opt. Soc. Amer. 46*, 362 (1956).

J-1 Jaffe, L., "Effect of polarized light on polarity of *Fucus*," *Science 123*, 1081 (1956).

J-2 Jaffe, L. F., "Tropistic responses of zygotes of the Fucaceae to polarized light," *Exptl. Cell Research 15*, 282 (1958).

J-3 Jamnik, D., and P. Axel, "Plane polarization of 15.1 Mev bremsstrahlung from 25-Mev electrons," *Phys. Rev. 117*, 194 (1960). A degree of polarization of 21 percent was found.

J-4 Jander, R., and T. H. Waterman, "Sensory discrimination between polarized light and light intensity patterns by arthropods," *J. Cellular Comp. Physiol. 56*, 137 (1960).

J-5 Janeschitz-Kriegl, H., "New apparatus for measuring flow-birefringence," *Rev. Sci. Instr. 31*, 119 (1960). Employs Glan-Thompson polarizer.

J-6 Jauch, J. M., and F. Rohrlich, *Theory of photons and electrons* (Addison-Wesley, Reading, Massachusetts, 1955). Presents quantum-mechanical approach to polarized light and the Stokes parameters.

J-7 Jehu, V. J., "Assessment of polarized headlighting," *Intern. Road Safety Traffic Rev. 4*, 26 (1956). Describes German and American systems; proposes use of mixed (polarized and unpolarized) beams.

J-8 Jelley, J. V., *Čerenkov radiation and its applications* (Pergamon, London, 1958).

J-9 Jenkins, F. A., and H. E. White, *Fundamentals of optics* (McGraw-Hill, New York, ed. 2, 1950).

J-10 Jensen, C., "Die Himmelsstrahlung," in H. Geiger and K. Scheel, ed., *Handbuch der Physik*, vol. 19 (Springer, Berlin, 1928), chap. 4.

J-11 Jerrard, H. G., "Optical compensators for measurement of elliptical polarization," *J. Opt. Soc. Amer. 38*, 35 (1948).

J-12 Jerrard, H. G., "Use of a half-shadow plate with uniform field compensators," *J. Sci. Instr. 28*, 10 (1951).

J-13 Jerrard, H. G., "The calibration of quarter-wave plates," *J. Opt. Soc. Amer. 42*, 159 (1952).

J-14 Jerrard, H. G., "Transmission of light through birefringent and optically active media: the Poincaré sphere," *J. Opt. Soc. Amer. 44*, 634 (1954).

J-15 Jessop, H. T., and F. C. Harris, *Photoelasticity principles and methods* (Dover, New York, 1950). 180 pp.

J-16 Jessop, H. T., "On the Tardy and Senarmont methods of measuring fractional relative retardation," *Brit. J. Appl. Phys. 4*, 138 (1953).

J-17 Jessop, H. T., "Photoelasticity," in S. Flügge, ed., *Encyclopedia of physics*, vol. 6 (Springer, Berlin, 1958).

J-18 Johannsen, A., *Manual of petrographic methods* (McGraw-Hill, New York, 1918). Contains descriptions of many kinds of birefringence polarizers.

J-19 Jones, R. C., "New calculus for the treatment of optical systems. I. Description and discussion of the calculus," *J. Opt. Soc. Amer. 31*, 488 (1941). For Part II, see Hurwitz and Jones, H-40.

J-20 Jones, R. C., "New calculus for the treatment of optical systems. III. The Sohncke theory of optical activity," *J. Opt. Soc. Amer. 31*, 500 (1941).

J-21 Jones, R. C., "New calculus for the treatment of optical systems. IV," *J. Opt. Soc. Amer. 32*, 486 (1942).
J-22 Jones, R. C., unpublished notes used in talk abstracted in J-23.
J-23 Jones, R. C., "Theory of sheet polarizers," *J. Opt. Soc. Amer. 35*, 803 (1945). Deals with density ratio, extent of parallelism of dichroic needles; a brief abstract.
J-24 Jones, R. C., "Theory of sheet polarizers." Unpublished manuscript no. 539 of Oct. 1945 showing relation between axial ratio and density ratio.
J-25 Jones, R. C., "New calculus for the treatment of optical systems. V. A more general formulation and description of another calculus," *J. Opt. Soc. Amer. 37*, 107 (1947).
J-26 Jones, R. C., "New calculus for the treatment of optical systems. VI. Experimental determination of the matrix," *J. Opt. Soc. Amer. 37*, 110 (1947).
J-27 Jones, R. C., "New calculus for the treatment of optical systems. VII. Properties of the N-matrices," *J. Opt. Soc. Amer. 38*, 671 (1948).
J-28 Jones, R. C., "On the possibility of a spathic polarizer which transmits more than one-half of the incident unpolarized light," *J. Opt. Soc. Amer. 39*, 1058 (1949).
J-29 Jones, R. C., and C. D. West, "On the properties of polarization elements as used in optical instruments. II. Sinusoidal modulators," *J. Opt. Soc Amer. 41*, 982 (1951).
J-30 Jones, R. C., "On reversibility and irreversibility in optics," *J. Opt. Soc. Amer. 43*, 138 (1953).
J-31 Jones, R. C., and W. A. Shurcliff, "Equipment to measure and control synchronization errors in 3-D projection," *J. Soc. Motion Picture Television Engrs. 62*, 134 (1954).
J-32 Jones, R. C., "Transmittance of a train of three polarizers," *J. Opt. Soc. Amer. 46*, 528 (1956).
J-33 Jones, R. C., "New calculus for the treatment of optical systems. VIII. Electromagnetic theory," *J. Opt. Soc. Amer. 46*, 126 (1956). Deals with N-matrices.
J-34 Jones, R. V., and J. C. S. Richards, "Polarization of light by narrow slits," *Proc. Roy. Soc. (London) A 225*, 122 (1954).
K-1 Kalmus, H., "Sun navigation by animals," *Nature 173*, 657 (1954).
K-2 Kalmus, H., "The sun navigation of animals," *Sci. Am.* (Oct. 1954), p. 74.
K-3 Käsemann, E., "Die Dichroismus des Zellulosefarbstoffkomplexes und seine technische Anwendung als Polarisationsfilter," *Optik 3*, 521 (1948).
K-4 Kaufman, I., "Band between microwave and infrared regions," *Proc. IRE 47*, 381 (1959).
K-5 Kaye, W., "Near-infrared spectroscopy. II. Instrumentation and technique," *Spectrochim. Acta 7*, 181 (1955).
K-6 Kelly, R. L., "Shift of photoconductive peak with polarization in CdS," *Bull. Am. Phys. Soc.* [2] *2*, 387 (1957). Abstract.
K-7 Kennedy, D., and E. R. Baylor, "Analysis of polarized light by the bee's eye," *Nature 191*, 34 (1961). Casts doubt on Autrum's hypothesis that analysis of the light occurs in the receptors themselves.
K-8 Kerker, M., "Use of white light in determining particle radius by the polarization ratio of the scattered light," *J. Colloid Sci. 5*, 165 (1950).

See also M. Kerker, and M. I. Hampton, article with same title, *J. Opt. Soc. Amer. 43*, 370 (1953).

K-9 Kerr, J., "A new relation between electricity and light: dielectrified media birefringent," *Phil. Mag.* [4] *50*, 337 (1875).

— Keston, A. S., U.S. patent 2,829,555 (1958). Polarimeter employing two beams (polarized in slightly different azimuths) and two photocells.

K-10 Keussler, V., and P. Manogg, "Über die Emission polarisierten Lichtes durch glühende Metalloberflächen und die darbei vorhandene räumliche Intensitätsverteilung," *Optik 17*, 602 (1960).

K-11 Klyne, W., and A. C. Parker, "Optical rotatory dispersion," in A. Weissberger, ed., *Physical methods in organic chemistry*, vol. 1, part 3 (Interscience, New York, ed. 3, 1960), chap. 34.

K-12 Koester, C. J., "Achromatic combinations of half-wave plates," *J. Opt. Soc. Amer. 49*, 405 (1959).

K-13 Koester, C. J., "Half-shade eyepieces for the A. O. Baker interference microscope," *J. Opt. Soc. Amer. 49*, 560 (1959).

K-14 Koester, C. J., H. Osterberg, and H. E. Willman, Jr., "Transmittance measurements with an interference microscope," *J. Opt. Soc. Amer. 50*, 477 (1960). Describes use of polarizers and retarders in interference microscopes.

K-15 Kondo, T., "Photoanisotropic effects in dyes," *Z. wiss. Phot. 31*, 153 (1932). Says that Weigert effect was found in 450 out of 1700 dyes.

K-16 Kossel, D., U.S. patent 2,809,555 (1957). "Light rays dividing system" for binocular microscope to provide equal intensity for both eyes, even for a polarizing specimen.

K-17 Kraft, C. L., "Broad band blue lighting system for radar approach control centers: Evaluations and refinements based on three years of operational use," Wright Air Development Center Tech. Rept. 56–71; Armed Services Technical Information Agency document No. AD 118090 (1956), 96 pp.

K-18 Kremers, H. C., "Optical silver chloride," *J. Opt. Soc. Amer. 37*, 337 (1947).

K-19 Kriebel, R. T., "Stereoscopic photography," *Complete Photographer 9*, 3308 (1943).

K-20 Krimm, S., C. Y. Liang, and G. B. B. M. Sutherland, "Infrared spectra of high polymers, V. Polyvinyl alcohol," *J. Polymer Sci. 22*, 227 (1956). An excellent review of the structure of PVA; deals with polarized infrared spectra and x-ray diffraction patterns.

K-21 Kubota, H., and K. Shimizu, "Experiment on the sensitive color," *J. Opt. Soc. Amer. 47*, 1121 (1957). Proposes means for getting a more sensitive color effect than is afforded by a conventional full-wave retarder between crossed polarizers. Employs a half-wave retarder between parallel polarizers.

K-22 Kuhn, R., "Synthesis of polyenes," *J. Chem. Soc. 1*, 605 (1938).

K-23 Kuscer, I., and M. Ribaric, "Matrix formalism in the theory of diffusion of light," *Optica Acta 6*, 42 (1959).

L-1 Lagemann, R. T., and T. G. Miller, "Thallium bromide-iodide (KRS-5) as an infra-red polarizer," *J. Opt. Soc. Amer. 41*, 1063 (1951).

L-2 Laine, P., "Sur les erreurs entraînées par l'inexactitude des lames demi-onde dans l'analyse des vibrations faiblement elliptiques et sur l'étalonnages des lames demi-onde et quart d'onde," *Compt. rend. 192*, 1215 (1931).

L-3 Lambe, J., and W. D. Compton, "Luminescence and symmetry properties of color centers," *Phys. Rev. 106*, 684 (1957). Discusses dichroism in KBr.

L-4 Land, E. H., "A new polarizer for light in the form of an extensive synthetic sheet," talk given at the Harvard Physics Colloquium, Cambridge, Massachusetts, Feb. 8, 1932.

L-5 Land, E. H., "Polaroid and the headlight problem," *J. Franklin Inst. 224*, 269 (1937).

L-6 Land, E. H., "Vectographs: Images in terms of vectorial inequality and their application to three-dimensional representation," *J. Opt. Soc. Amer. 30*, 230 (1940).

L-7 Land, E. H., "Polarized light in the transportation industries," University of Michigan Official Publication *42*, No. 42, 1940. (Michigan-Life Conference on New Technologies in Transportation, p. 151.)

L-8 Land, E. H., and C. D. West, "Dichroism and dichroic polarizers," in J. Alexander, ed., *Colloid chemistry* (Reinhold, New York, 1946), vol. 6, chap. 6.

L-9 Land, E. H., *The completion of the technical development stage of the Polaroid glare-eliminating headlight system* (pamphlet prepared for presentation at a Nov. 10, 1947, meeting of the American Association of Motor Vehicles Administrators), 23 pages, 17 illustr.

L-10 Land, E. H., *The Polaroid headlight system* (Highway Research Board Bulletin No. 11, Division of Engineering and Industrial Research, National Research Council, Washington, 1948), 20 pp.

L-11 Land, E. H., "Polarized headlights for safe night driving," *Traffic Quarterly* (Eno Foundation for Highway Traffic Control, Saugatuck, Connecticut; October 1948), 11 pp.

L-12 Land, E. H., and L. W. Chubb, Jr., "Polarized light for auto headlights," *Traffic Eng. Mag.* (April and July 1950).

L-13 Land, E. H., "Some aspects of the development of sheet polarizers," *J. Opt. Soc. Amer. 41*, 957 (1951).

— Land, E. H., U.S. patents:

1,918,848 (1933), with J. S. Friedman. Polarizer containing aligned crystals of herapathite.

1,951,664 (1934). Suspensions of herapathite, etc.

1,955,923 (1939). Light valve employing controllable herapathite crystals.

1,956,867 (1934). Polarizer employing a periodide.

1,963,496 (1934). Light valve employing controllable particles.

1,989,371 (1935). Extrusion method of orienting objects.

2,005,426 (1935). Variable-density sunglasses.

2,011,553 (1935). Stretching method of orienting objects.

2,018,214 (1935). Advertising display employing polarizers, etc.

2,018,963 (1935). Suppression of specularly reflected glare by means of linear or circular polarizers.

2,031,045 (1936). Headlight system.

2,041,138 (1936). Flow method of orienting objects.

2,078,181 (1937). Polarizer system for microscope.

2,078,254 (1937). Polarizer containing fine, nonscattering crystals.

2,079,621 (1937). Polarizer system for microscope used in examining opaque objects.

2,084,350 (1937). 3-D viewer for viewing a pair of nearby, large-area pictures; employs a large semitransparent mirror at 45°.
2,096,696 (1937). Reading lamp employing two polarizers.
2,099,694 (1937). Circular polarizer of two-layer type, and use thereof in visors or viewers for headlight systems or 3-D projection systems.
2,102,632 (1937). Automobile visor adjustable (by interposing a retarder) to either of two unlike functions, namely reducing reflection from road or dimming of coded light from oncoming car's headlight.
2,106,752 (1938). Prism-type beam splitter used with polarizers in taking or projecting 3-D photographs.
2,122,178 (1938). Diffusing polarizer.
2,123,901 (1938). Diffusing polarizer producing two useful components.
2,123,902 (1938). Diffusing polarizer producing a specular component and a cylindrically spread component.
2,146,962 (1938). Dynamic display device employing polarizers and retarders.
2,158,129 (1939). Dynamic display device to be affixed to opaque surface.
2,158,130 (1939). Diffusing polarizer.
2,165,973 (1939). Polarizer containing very small crystals of herapathite in very low volume concentration.
2,168,220 (1939). Polarizing safety glass.
2,168,221 (1939). Laminated polarizer to be used close to a hot body.
2,173,304 (1939), with H. G. Rogers. Relates to K-sheet.
2,174,269 (1939). Photoelastic analysis apparatus employing polarizers and a variable achromatic retarder.
2,174,270 (1939). Display device employing reflection from surface of birefringent sheet.
2,178,996 (1939). Polarizer employing small crystals of a sulfate of an alkaloid.
2,180,113 (1939). Translucent screen employing immiscible polymers having different indices.
2,180,114 (1939). Headlight polarizer assembly having efficiency exceeding 50 percent.
2,184,999 (1939). Color filter employing several polarizers and retarders.
2,185,000 (1939). Headlight polarizer system making use of a specular and a diffuse component.
2,200,959 (1940). Display system involving showcase windows.
2,203,687 (1940), with J. Mahler. Vectograph system.
2,204,604 (1940). Images in terms of polarizance, with the aid of "resists."
2,212,880 (1940). Prism-type beam splitter used in 3-D projection.
2,237,565 (1941). Polarizing visor containing adjustable retarder to compensate for windshield's retardance.
2,237,566 (1941). Variable-density window employing polarizers and an adjustable retarder.
2,237,567 (1941). Relates to H-sheet.
2,252,324 (1941). Incandescent lamp coated with polarizing material.
2,255,933 (1941). Variable-density window employing polarizers and an adjustable retarder consisting of a rubbery layer in shear between the polarizers.
2,256,093 (1941). Method of fabricating a retarding coating.

2,270,323 (1942), with C. D. West. PVA film having high birefringence.
2,270,535 (1942), with C. J. T. Young. Automobile-headlight polarizer employing oblique surfaces and having efficiency exceeding 50 percent.
2,281,100 (1942). Orienting polarizing particles on a softened surface.
2,281,101 (1942). Reflection-type vectograph.
2,287,556 (1942). Diffusing screen employing incompatible polymers and conserving polarization.
2,289,712 (1942), with C. D. West. Discusses making herapathite crystals and polarizers employing them.
2,289,713 (1942). Method of making J-type polarizer on a curved support by means of stroking.
2,289,714 (1942). Colored vectograph images.
2,289,715 (1942). Composite films for receiving vectograph images.
2,298,058 (1942). Variable-hue sunglasses.
2,298,059 (1942). Variable-hue filter for camera.
2,299,906 (1942). Light-sensitive, layered material for vectograph.
2,302,613 (1942). Polarizing desk lamp.
2,306,108 (1942), with H. G. Rogers. Method of making K-sheet.
2,311,840 (1943). Variable-density window employing two polarizers, one of which can be rotated or moved out of the way.
2,313,349 (1943). Variable-density window employing adjustable retarder.
2,315,373 (1943). Process of making a vectograph print.
2,319,816 (1943). Polarizer consisting of stretched glass that contains a reduced metal.
2,323,059 (1943). Colored wall panels employing polarizers and retarders.
2,328,219 (1943). H-sheet based on vinyl compounds.
2,329,543 (1943). Polarizing images produced by destroying the polarizance in certain regions.
2,334,418 (1943). Use of circular polarizers in traffic-signal light.
2,343,775 (1944). Polarizer manufacture employing extrusion and evaporation.
2,346,766 (1944). Polarizer containing aligned fibers in which aligned dichroic molecules are present.
2,348,912 (1944). Self-analyzing dichroic image.
2,356,250 (1944). Adhesive for use in polarizers.
2,356,251 (1944). Polarizer employing birefringent needles.
2,356,252 (1944). Shatterproof polarizer.
2,359,428 (1944). Aligning needle-like crystals by means of stretching, smearing, or rolling operations.
2,362,832 (1944). Remote-communication or -control system employing light beams of varying polarization form.
2,373,035 (1945). Polarizing image, as in vectograph.
2,376,493 (1945), with M. Grabau. Apparatus for generating sound waves controllable by means of polarizers.
2,380,363 (1945), with R. P. Blake. Means of orienting the surface of a PVA sheet.
2,397,149 (1946). Producing opposite orientations on opposite faces of a PVA sheet to be used in vectograph.
2,397,272 (1946). Vectograph identification badge.

2,397,273 (1946). Range finder employing polarizers.
2,397,276 (1946). Vectograph system employing materials having opposite sign of refraction indicatrix.
2,402,166 (1946). Vectograph sheet.
2,407,306 (1946). Range finder employing polarizers.
2,416,528 (1947). 3-D motion-picture viewers integral with theater ticket.
2,420,252 (1947). Optical ring sight.
2,420,253 (1947). Optical ring sight that includes a biaxial element.
2,423,503 (1947). Vectograph sheet.
2,423,504 (1947). Vectograph photography employing silver and iodine.
2,431,942 (1947). 3-D motion-picture viewers employing retarders and usable either side around.
2,431,943 (1947), with J. R. Swanton and J. W. Gibson. Press used in manufacturing polarizer sheet.
2,440,102 (1948). Producing two-tone polarizing images.
2,440,103 (1948). Polarizer protected by laminated glass plates.
2,440,104 (1948). 3-D motion-picture viewers made by a folding process.
2,440,105 (1948). Producing polarizing and nonpolarizing images in a single picture.
2,440,106 (1948). Producing polarizing images.
2,445,581 (1948). H-sheet production method employing boric acid.
2,454,515 (1948). H-sheet basic patent. Calls for a rubber-elastic base and an added dichromophore.
2,458,179 (1949). Headlight system employing polarizer at 35°.
2,493,200 (1950). Variable-hue filter for use in producing color television. Employs several spectrally selective dichroic polarizers and three electro-optic retarders.
2,547,763 (1951). Method of stretching sheet.
2,788,707 (1957). Calcite parfocalizing plate for vectograph.

L-14 Langsdorf, A., Jr., and L. A. DuBridge, "Optical rotation of unpolarized light," *J. Opt. Soc. Amer. 24*, 1 (1934).

L-15 Laurence, J., "Reflection characteristics with polarized light," *J. Opt. Soc. Amer. 31*, 9 (1941).

L-16 Lawrence, A. S. C., in *Thorpe's dictionary of applied chemistry* (Longmans, Green, New York, vol. 7, 1946), p. 350. Discusses mesomorphic (smectic and nematic) states.

L-17 Le Fèvre, C. G., and R. J. W. Le Fèvre, "The Kerr effect," in A. Weissberger, ed., *Physical methods in organic chemistry*, vol. 1, part 3 (Interscience, New York, ed. 3, 1960), chap. 36.

L-18 Lenoble, J., "État de polarisation du rayonnement diffusé dans les milieux naturels (mer et atmosphere)," *J. phys. radium 18*, 47 S (1957). Employs the Stokes vector and the matrices of scatterers.

L-19 LeRoux, P., "Étude du pléochroïsime du spath d'Islande dans le spectre infrarouge," *Compt. rend. 196*, 394 (1933).

L-20 Lester, H. M., and O. W. Richards, "Stereo photomicrography with cameras of fixed interocular distance," *Photo. Eng. 5*, 149 (1954). Discusses not only 3-D photography but also the adapting of mono-objective, dual-ocular microscopes to 3-D.

L-21 Leven, M. M., "Quantitative three-dimensional photoelasticity," *Proc. Soc. Exp. Stress Anal. 12*, 157 (1955).

L-22 Linhart, J. G., "Cherenkov radiation," *Research 8*, 402 (1955).

L-23 Lodge, A. S., "Network theory of flow birefringence and stress in concentrated polymer solutions," *Trans. Faraday Soc. 52*, 120 (1956).

L-24 Loferski, J. J., "Optical polarization in single crystal of tellurium," *Phys. Rev. 87*, 905 (1952).

L-25 Locquin, M., *Bull. microscop. appl. 6*, 33 (1956). Reviews refractoanisotropy of biological materials.

L-26 Lostis, M. P., "Étude et réalisation d'une lame demi-onde en utilisant les propriétés des couches minces," *J. phys. radium 18*, 51 S (1957). Achromatic retarder.

L-27 Lowry, T. M., *Optical rotatory power* (Longmans, Green, London, 1935).

L-28 Lyot, B. F., "Un monochromator à grand champ utilisant les interférences en lumière polarisée," *Compt. rend. 197*, 1593 (1933).

L-29 Lyot, B. F., "Le filtre monochromatique polarisant et ses applications en physique solaire," *Ann. astrophys. 7*, 31 (1944).

— Lyot, B. F., U.S. patent 2,718,170. Slitless spectrophotometer.

M-1 McDermott, M. N., and R. Novick, "Large-aperture polarizers and retardation plates for use in the far ultraviolet," *J. Opt. Soc. Amer. 51*, 1008 (1961). Deals with dichroic polarizers useful in the range from 215 to 400 mμ, and with retarders of polyvinyl alcohol and mica.

M-2 McMaster, W. H., "Polarization and the Stokes parameters," *Am. J. Phys. 22*, 351 (1954).

M-3 McMaster, W. H., "Matrix representation of polarization," *Revs. Mod. Phys. 33*, 8 (1961).

M-4 McMaster, W. H., and F. L. Hereford, "Angular distribution of photoelectrons produced by 0.4–0.8-Mev polarized photons," *Phys. Rev. 95*, 723 (1954).

M-5 McMaster, W. H., *Matrix representation of polarization* (Report UCRL 5496, University of California Radiation Laboratory, 1959); also *Revs. Mod. Phys. 33*, 8 (1961).

M-6 McNally, J. G., and S. E. Sheppard, "Double refraction in cellulose acetate and nitrate films," *J. Phys. Chem. 34*, 165 (1930). Describes improved polariscope. Says cellulose acetate, for example, can be isotropic, uniaxial, or biaxial depending on how it was supported during drying.

M-7 Madden, R. P., "10 to 15 micron thick evaporated silver films for infrared gratings," paper delivered to the spring meeting of the Optical Society of America, April 8, 1955. Dealt with polarization produced by reflection gratings made of silver.

M-8 Maden, H. G., "On a modification of Foucault's and Ahrens's prisms," *Nature 31*, 371 (1884–85). Polarizing prism suitable for transmitting polarized-light images that are almost free of deviation and dispersion.

— Mahler, J., U.S. patent 2,674,156. Scheme for reducing ghosts in 3-D images.

M-9 Makas, A. S., and W. A. Shurcliff, "New arrangement of silver chloride polarizer for the infrared," *J. Opt. Soc. Amer. 45*, 998 (1955).

M-10 Malcolm, B. R., and A. Elliott, "Sensitive photoelectric polarimeter," *J. Sci. Instr. 34*, 48 (1957). Precision of 0.001° is claimed.

M-11 Mallemann, R. de., "Constantes selectionnées pouver rotatoire magnétique (effet Faraday)," in *Tables de constantes et données numériques* (Herman, Paris, 1951; published for the International Union for Pure and Applied Chemistry), pt. 3.

M-12 Malus, E., *Mém soc. Arcueil 1*, 113 (1808). Announces discovery of Malus's cosine-squared law.

M-13 Manchester, H., "The magic crystal," a chapter from *New world of machines* (Random House, New York, 1945). Popular discussion of polarizers and their applications.

M-14 Mariot, M. L., "Polarisation de la lumière en relativité générale," *J. phys. radium 21*, 80 S (1960).

M-15 Marks, A. M., "Multilayer polarizers and their application to general polarized lighting," *Illum. Eng. 54*, 123 (1959).

— Marks, A. M., U.S. patents:

2,104,949 (1938). Optical sheet on which a continuous layer of optically active substance has been formed.

2,199,227 (1940). Evaporation method of producing crystalline films.

2,777,011 (1957). Three-dimensional display system employing a special thick screen.

M-16 Meecham, W. C., and C. W. Peters, "Reflection of plane-polarized, electromagnetic radiation from an echelette diffraction grating," *J. Appl. Phys. 28*, 216 (1957).

M-17 Meier, R., and H. H. Günthard, "Germanium polarizers for the infrared," *J. Opt. Soc. Amer. 49*, 1122 (1959).

M-18 Mesnager, M., "Sur la determination optique des tensions intérieures dans les solides a trois dimensions," *Compt. rend. 190*, 1249 (1930).

M-19 Mielenz, K. D., and R. C. Jones, "Die Eignung von Polarisations-filtern für photometrische Messungen," *Optik 15*, 656 (1958).

M-20 Mindlin, R. D., "A reflection polariscope for photoelastic analysis," *Rev. Sci. Instr. 5*, 224 (1934).

M-21 Mitchell, S., and J. Veitch, "Rotary dispersion measurements with Unicam spectrophotometer," *Nature 168*, 662 (1951). Describes a spectropolarimeter.

M-22 Mitchell, S., "Accessories for measuring circular dichroism and rotatory dispersion with a spectrophotometer," *J. Sci. Inst. 34*, 89 (1957). Covers 3000–10,000 A range.

M-23 Mitsuishi, A., Y. Yamada, S. Fujita, and H. Yoshinaga, "Polarizer for the far-infrared region," *J. Opt. Soc. Amer. 50*, 433 (1960). Device consists of 15 layers of thin polyethylene films and performs well in the range from 3 to 200 μ.

M-24 Mooney, F., "A modification of the Fresnel rhomb," *J. Opt. Soc. Amer. 42*, 181 (1952).

M-25 Motz, H., W. Thon, and R. N. Whitehurst, "Experiments on radiation by fast electron beams," *J. Appl. Phys. 24*, 826 (1953). Describes the Motz generator.

M-26 Mueller, H., "Memorandum on the polarization optics of the photoelastic shutter," Report No. 2 of the OSRD project OEMsr-576, Nov. 15, 1943. A declassified report.

M-27 Mueller, H., informal notes of about 1943 on Course 8.26 at Massachusetts Institute of Technology.

M-28 Mueller, H., "The foundations of optics," *J. Opt. Soc. Amer. 38*, 661, 1948. Deals with Stokes parameters and matrices.

— Mueller, H., U.S. patent 2,707,749. System of light beam communication. Involves two beams modulated differently in terms of polarization.

M-29 Muralt, A. von, and J. T. Edsall, "Studies in the physical chemistry of muscle globulin. III. The anisotropy of myosin and the angle of isocline. IV. The anisotropy of myosin and double refraction of flow," *J. Biol. Chem. 89*, 315, 351 (1930).

M-30 Mussett, E. A., "On refractive index determinations by Brewster angle measurements," *J. Opt. Soc. Amer. 46*, 369 (1956).

N-1 Nathan, A. M., "Polarization technique for improving visibility in fog and haze," *J. Opt. Soc. Amer. 48*, 285 (1958). Linear or circular polarizers can cause five- to fiftyfold improvement.

N-2 Newman, R., and R. S. Halford, "An efficient, convenient polarizer for infra-red radiation," *Rev. Sci. Instr. 19*, 270 (1948).

N-3 Ney, E. P., W. F. Huch, R. W. Maas, and R. B. Thorness, "Eclipse polarimeter," *Astrophys. J. 132*, 812 (1960). Employs a rotating disk containing many small windows covered with HN-32 and HR polarizers at a variety of azimuths.

N-4 Nicholson, W. Q., and I. Ross, "Kerr-cell shutter has submicrosecond speed," *Electronics 28*, 171 (June 1955).

N-5 Nicol, W., "On a method of so far increasing the divergence of the two rays in calcareous-spar that only one image may be seen at a time," *Edinburgh New Phil. J. 6*, 83 (1828, 1829, 1833).

N-6 Nikitine, S., "Généralisation de la théorie du photodichroïsme," *Compt. rend. 207*, 331 (1938).

N-7 Nikitine, S., "Sur l'anistropie d'absorption de différentes radiations pour les molécules de quelques colorants photosensibles," *Compt. rend. 208*, 805 (1939).

O-1 Öhman, Y., "A new monochromator," *Nature 141*, 291 (1938).

O-2 Optical Society of America, Committee on Colorimetry, *The science of color* (Crowell, New York, 1953).

O-3 Ore, A., "Entropy of radiation," *Phys. Rev. 98*, 887 (1955). Recalls Planck's derivation of an entropy equation for radiation.

O-4 Osipov-King, W. A., "Über eine neue Konstruktion von Polarisationsprismen," *Compt. rend. acad. sci. U.R.S.S. 4* (No. 2), 53 (1936).

O-5 Oster, G., and A. W. Pollister, *Physical techniques in biological research* (Academic Press, New York, 1955).

P-1 Pancharatnam, S., "Achromatic combinations of birefringent plates. Part I. An achromatic circular polarizer," *Proc. Indian Acad. Sci. A 41*, 130 (1955).

P-2 Pancharatnam, S., "Achromatic combinations of birefringent plates. Part II. An achromatic quarter-wave plate," *Proc. Indian Acad. Sci. A 41*, 137 (1955).

P-3 Pancharatnam, S., "Propagation of light in absorbing biaxial crystals: II. Experimental," *Proc. Ind. Acad. Sci. 42*, 235 (1955).

P-4 Pancharatnam, S., "Generalized theory of interference and its applications: Part 2. Partially coherent pencils," *Proc. Ind. Acad. Sci. 44*, 398 (1956).

P-5 Pancharatnam, S., "Light propagation in absorbing crystals possessing optical activity," *Proc. Indian Acad. Sci. 46*, 280 (1957). Discusses non-orthogonality of eigenvectors of absorbing, retarding bodies.

P-6 Parke, N. G., III, "Matrix optics," Ph.D. thesis, Department of Physics, Massachusetts Institute of Technology (May 1, 1948), 181 pp.

P-7 Parke, N. G., III, "Matrix algebra of electromagnetic waves," Technical Report No. 70, Research Laboratory of Electronics, Massachusetts Institute of Technology (June 30, 1948), 28 pp. Discusses Jones calculus, Mueller calculus, and Wiener algebra.

P-8 Parke, N. G., III, "Statistical optics. I. Radiation," Technical Report No. 95, Research Laboratory of Electronics, Massachusetts Institute of Technology (January 31, 1949), 15 pp. Shows how to proceed from the Jones calculus to the Mueller calculus.

P-9 Parke, N. G., III, "Statistical optics. II. Mueller phenomenological algebra," Technical Report No. 119, Research Laboratory of Electronics, Massachusetts Institute of Technology (June 15, 1949). Describes and relates the Jones calculus and the Mueller calculus.

P-10 Partington, J. R., *An advanced treatise on physical chemistry*, vol. 4, "Physico-chemical optics" (Longmans, Green, New York, 1953), 688 pp. Detailed discussion of birefringence, optical activity, dichroism.

P-11 Perrin, F., "La fluorescence des solutions," *Ann. Phys. 12*, 169 (1929). Describes a method of calculating fluorescence decay time in terms of the loss in degree of polarization, the viscosity, etc., assuming that the exciting radiation was 100-percent linearly polarized.

P-12 Perrin, Francis, "Polarization of light scattered by isotropic opalescent media," *J. Chem. Phys. 10*, 415 (1942). Discusses the Stokes vector and a new calculus.

P-13 Perry, C. C., "Visual flow analysis," *Product Eng. 26*, 154 (1955). Shows photographs of "flow through a convergent-divergent nozzle as indicated by the streaming double refraction system using a synthetic dye solution."

P-14 Peterlin, A., and H. A. Stuart, *Doppelbrechung insbesonders kunstliche Doppelbrechung* (Akademische Verlag, Leipzig, 1943), 115 pp.; republished in 1948 by J. W. Edwards, Ann Arbor, Michigan. Discusses birefringence produced by flow, electric fields, magnetic fields, etc.

P-15 Pfund, A. H., "Improved infra-red polarizer," *J. Opt. Soc. Amer. 37*, 558 (1947).

P-16 Piddington, J. H., "Cosmical electrodynamics," *Proc. I.R.E. 46*, 349 (1958). Discusses polarized light from Crab Nebula.

P-17 Planck, M., *Theorie der Warmestrahlung* (Barth, Leipzig, ed. 1, 1906).

P-18 Planck, M., *Theory of heat radiation* (Blakiston, Philadelphia, 1914).

P-19 Pockels, F., *Lehrbuch der Kristalloptik* (Teubner, Leipzig and Berlin, 1906).

P-20 Poincaré, H., Théorie mathématique de la lumière (Gauthiers-Villars, Paris, 1892).

P-21 Polacoat, Inc., Bulletin p105 of Nov. 1, 1957.

P-22 Polaroid Corporation, *Polarized light and its applications* (booklet, 1945), 48 pp. Revision of M. Grabau's earlier version; see G-12.

P-23 Polaroid Corporation, *How to make Polaroid 3-D vectographs; preliminary instruction manual* F54 (pamphlet issued by Polaroid Corp. in 1940 or early 1941).

P-24 Polaroid Corporation, "Show me the way to go home by polarized light," *Polaroid Reporter*, No. 1 (1951).

P-25 Polaroid Corporation, *Polaroid vectograph film, supplementary information* (Pamphlet F-1380, May 1954), 4 pp.

P-26 Polaroid Corporation, *Photometric specifications of Polaroid Corporation sheet polarizers* (pamphlet dated Oct. 10, 1954).

P-27 Polaroid Corporation, *The application of polarized light to basic design problems* (Pamphlet F-2165, 1959), 12 pp.

P-28 Pollak, L. W., and H. Wilhelm, "Über die Verwendung von Flächenpolarisatoren in der meteorologischen Optik," *Zeiss Nachr.*, p. 307 (January 1939).

P-29 Porter, C. S., E. G. Spencer, and R. C. LeCraw, "Transparent ferromagnetic light modulator using yttrium iron garnet," *J. Appl. Phys. 29*, 495 (1958).

P-30 Preston, T., *The theory of light* (Macmillan, London, ed. 5, 1928).

P-31 Prishivalko, A. P., "Determination of the optical constants of absorbing substances from the measurements of Stokes' parameters of reflected light," *Optics and Spectroscopy 9*, 256 (1960).

P-32 Pritchard, B. S., and H. R. Blackwell, "Preliminary studies of visibility on the highway in fog," University of Michigan Engineering Research Institute, Report 2557-2-F, July 1957.

P-33 Provostaye, F., and P. Desains, "Mémoire sur la polarisation de la chaleur par réfraction simple," *Ann. chim. et phys. 30*, 158 (1850).

R-1 Ramachandran, G. N., and S. Ramaseshan, "Magneto-optic rotation in birefringent media. Application of the Poincaré sphere," *J. Opt. Soc. Amer. 42*, 49 (1952).

R-2 Ramachandran, G. N., and S. Ramaseshan, "Crystal optics," a section in S. Flügge, ed., *Encyclopedia of Physics*, vol. 25/1 (Springer, Berlin, 1961). Includes discussions of the Poincaré sphere and the Jones calculus.

R-3 Randall, D. D., "New photoelectric method for the calibration of retardation plates," *J. Opt. Soc. Amer. 44*, 600 (1954).

R-4 Ravilious, C. F., R. T. Farrar, and S. H. Liebson, "Measurement of organic fluorescence decay times," *J. Opt. Soc. Amer. 44*, 238 (1954). Determines fluorescence decay time in terms of polarization defect of the fluorescent light, assuming that the exciting light is 100-percent polarized.

R-5 Rich, A., "Use of the Senarmont compensator for measuring double refraction of flow," *J. Opt. Soc. Amer. 45*, 393 (1955).

R-6 Richards, O. W., *A. O. Baker interference microscope Model 7; Reference manual* (pamphlet; American Optical Co., 1958), 32 pp.

R-7 Richartz, M., and H. Hsu, "Analysis of elliptical polarization," *J. Opt. Soc. Amer. 39*, 136 (1949).

R-8 Richartz, M., "On the measurement of skylight polarization," *J. Opt. Soc. Amer. 50*, 302 (1960). Compares the visual methods available

R-9 Rinne, F., and M. Berek, *Anleitung zu optischen Untersuchungen mit dem Polarisationsmikroskop* (Schweizerbart'sche Verlagsbuchhandlung, Stuttgart, ed. 2, 1953), 366 pp.

R-10 Roberts, S., "Interpretation of the optical properties of metal surfaces," *Phys. Rev. 100*, 1667 (1955).

— Roehrig, J. R., U.S. patent 2,824,487. Apparatus for grading anisotropic fibers.

— Rogers, H. G., U.S. patent 2,255,940. Orientation method involving shrinking and restretching, as of K-sheet.

R-11 Roper, V., and K. D. Scott, "Seeing with polarized headlamps," *Illum. Eng. 36*, 1205 (1941).

R-12 Rosen, P., "Entropy of radiation," *Phys. Rev. 96*, 555 (1954).

R-13 Rossi, B., *Optics* (Addison-Wesley, Reading, Massachusetts, 1957), 510 pp.

R-14 Rothen, A., and M. Hanson, "Optical properties of surface films, II," *Rev. Sci. Instr. 20*, 66 (1949).

R-15 Rothen, A., "Improved method to measure the thickness of thin films with a photoelectric ellipsometer," *Rev. Sci. Instr. 28*, 283 (1957).

R-16 Rudolph, H., "Photoelectric polarimeter attachment," *J. Opt. Soc. Amer. 45*, 50 (1955). Contains an excellent discussion of accuracy, and an extensive bibliography.

— Ryan, W. H., U.S. patents:
2,263,684 (1941). Polarizers in variable-hue filter for darkroom.
2,811,893 (1957). Deals with methods of compensating for ghost images in 3-D presentations.

— Sage, S. J., U.S. patent 2,834,254. Electronic color filter.

S-1 Schawlow, A. L., and C. H. Townes, "Infrared and optical masers," *Phys. Rev. 112*, 1940 (1958). Mentions production of polarized light by masers.

S-2 Scheraga, H., and R. Signer, "Streaming birefringence," in A. Weissberger, ed., *Physical methods of organic chemistry*, vol. 1, part 3 (Interscience, New York, ed. 3, 1960), chap. 35.

S-3 Scherer, H., "Dichroitisch angefärbte Polarisatoren," *Z. Naturforsch. 6a*, 440 (1951).

S-4 Schick, L. H., and S. C. Miller, "Compton cross section for circular photon polarization and arbitrary electron spin orientation," *Bull. Am. Phys. Soc.* [2] *2*, 312 (1957).

S-5 Schmidt, W. J., "Polarisationsoptische analyse des Submikroskopischen Baues von Zellen und Geweben," in *Handbuch der biologischen Arbeitsmethoden*, Abt. 5, Teil 10, Heft 3 (1934), pp. 435–665.

S-6 Schöne, Herman, and Hedwig Schöne, "Eyestalk movements induced by polarized light in the ghost crab, *Ocypode quadrata*," *Science 134*, 675 (1961).

S-7 Schubert, F., "Optische Eigenschaften glühender Metalle," *Ann. Physik 29*, 473 (1937).

S-8 Sekera, Z., "Polarization of skylight," in S. Flügge, ed., *Encyclopedia of physics*, vol. 48 (Springer, Berlin, 1957).

S-9 Sekera, Z., "Light scattering in the atmosphere and the polarization of sky light," *J. Opt. Soc. Amer. 47*, 484 (1957).

S-10 Shklovsky, I. S., *Cosmic radio waves* (Harvard University Press, Cambridge, Mass., 1960). Includes discussion of polarization of light from Crab nebula.

— Short, F., U.S. patent 1,734,022 (1921). Polarizing headlight system using reflection polarizers.

S-11 Shurcliff, W. A., "Screens for 3-D and their effect on polarization," *J. Soc. Motion Picture Television Engrs. 62*, 125–133 (1954).

S-12 Shurcliff, W. A., "Polarized light," *Encyclopedia Americana* (Americana, New York, 1956).

S-13 Shurcliff, W. A., "Haidinger's brushes and circularly polarized light," *J. Opt. Soc. Amer. 45*, 399 (1955).

S-14 Shurcliff, W. A., "Circular polarizer improves viewing," *Electronics Design 4*, April 1, 1956.

S-15 Shurcliff, W. A., section on polarizing filters in *American Institute of Physics handbook* (McGraw-Hill, New York, 1957).

S-16 Simpson, J. A., H. W. Babcock, and H. D. Babcock, "Association of a unipolar magnetic region on the sun with changes of primary cosmic ray intensity," *Phys. Rev. 98*, 1402 (1955).

S-17 Skinner, C. A., "A universal polarimeter," *J. Opt. Soc. Amer. 10*, 491 (1925).

S-18 Sloan, L. L., "The Haidinger brush phenomenon," *J. Opt. Soc. Amer. 45*, 402 (1955).

S-19 Smartt, R. N., and W. H. Steel, "Birefringence of quartz and calcite," *J. Opt. Soc. Amer. 49*, 710 (1959).

S-20 Smith, F. H., "Microscopic interferometry," *Research 8*, 385 (1955).

S-21 Smith, S. J., and E. M. Purcell, "Visible light from localized surface charges moving across a grating," *Phys. Rev. 92*, 1069 (1953).

S-22 Šolc, I., "Further investigations on the birefracting filter" [in Russian], *Czechoslov. J. Phys. 5*, 80 (1955). Describes narrow-passband filter employing a large number of linear retarders.

S-23 Šolc, I., "Chain birefringent filters," *Czechoslov. J. Phys. 9*, 237 (1959). Describes a 14-plate filter having a 2-A transmission band.

S-24 Soleillet, P., "Sur les paramètres caractérisant la polarisation partielle de la lumière dans les phénomènes de fluorescence," *Ann. phys. 12*, 23 (1929). Discusses Stokes parameters and matrices.

S-25 Spence, J., "Optical anisotropy and the structure of cellulosic sheet material," *J. Phys. Chem. 43*, 865 (1939).

— Stadler, A. E. K., U.S. patent 2,527,593 (1950). Variable-hue filter employing two polarizers, a 90° retarder, and a 180° retarder.

S-26 Stamm, R. W., "The Polaroid stereoscope," *Am. J. Roentgenol. Radium Therapy 45*, 744 (1941). Shows how to view two x-ray photographs to obtain a 3-D effect.

S-26a Steel, W. H., R. N. Smartt, and R. G. Giovanelli, "A 1/8 Å birefringent filter for solar research," *Australian J. Phys. 14*, 201 (1961).

S-27 Stein, R. S., "Optical properties of oriented polystyrene," *J. Appl. Phys. 32*, 1280 (1961). Describes methods of calculating the birefringence and infrared dichroism.

S-28 Stokes, G. G., "On Haidinger's brushes" (1850), reprinted in *Mathematical and physical papers*, vol. 2 (Cambridge University Press, Cambridge, England, 1883), p. 362.

S-29 Stokes, G. G., "On the composition and resolution of streams of polarized light from different sources," *Trans. Cambridge Phil. Soc. 9*, 399 (1852); *Mathematical and physical papers*, vol. 3 (Cambridge University Press, Cambridge, England, 1901), p. 233.

S-30 Stokes, G. G., "On the intensity of the light reflected from or transmitted through a pile of plates," *Proc. Roy. Soc. (London) 11*, 545 (1862); *Mathematical and physical papers*, vol. 4 (Cambridge University Press, Cambridge, England, 1904), p. 145.

S-31 Strong, J., *Concepts of classical optics* (Freeman, San Francisco, 1958), 692 pp. Contains appendixes by G. F. Hull, Jr., on microwaves and by A. C. S. van Heel on Savart plates.

S-32 Sultanoff, M., "A 0.1 microsecond Kerr-cell shutter," *Phot. Eng. 5*, 80 (1954).

S-33 Swann, M. M., and J. M. Mitchison, "Refinements in polarized light microscopy," *J. Exptl. Biol. 27*, 226 (1950).

T-1 Tadokoro, H., S. Seki, and I. Nitta, "Infrared absorption spectrum of deuterated polyvinyl alcohol film," *J. Chem. Phys. 23*, 1351 (1955).

T-2 Takasaki, H., "Photoelectric measurement of polarized light by means of an ADP polarization modulator. I. Photoelectric polarimeter," *J. Opt. Soc. Amer. 51*, 462 (1961). Device employs a Senarmont compensator in which an ADP crystal excited by alternating current is incorporated.

T-3 Theophanis, G. A., "A Kerr-cell camera with synchronized light source for millimicrosecond reflected light photography," *J. Soc. Motion Picture and Television Engrs. 70*, 522 (1961). Employs three polarizers and a 60,000-v square-wave pulse to achieve 0.05-μs exposure time.

T-4 Thiessen, G. von, "Polarization des Lichtes beim Durchgang durch Metallspalte," *Optik 2*, 266 (1947).

T-5 Thompson, S. P., "On the Nicol prism and its modern varieties," in *Proceedings of the optical convention of 1905* (Williams and Norgate, London, 1905), pp. 216–235.

T-6 Thorpe, W. H., "Orientation and methods of communication of the honey bee and its sensitivity to the polarization of the light," *Nature 164*, 11 (1949).

T-7 Tinkham, M., and M. W. P. Strandberg, "The excitation of circular polarization in microwave cavities," *Proc. I.R.E. 43*, 734 (1955).

T-8 Townes, C. H., "Microwave spectroscopy," in M. H. Shamos and G. M. Murphy, eds., *Recent advances in science* (Interscience, New York, 1956).

T-9 Trentini, G. V., "Maximum transmission of electromagnetic waves by a pair of wire gratings," *J. Opt. Soc. Amer. 45*, 883 (1955).

T-10 Tsurumi, I., "Optical method for determining the small phase retardation with white light," *J. Opt. Soc. Amer. 45*, 1021 (1955).

T-11 Tuckerman, L. B., "On the intensity of the light reflected from or transmitted through a pile of plates," *J. Opt. Soc. Amer. 37*, 818 (1947).

T-12 Tutton, A. E. H., *Crystallography and practical crystal measurement* (Macmillan, London, 1911). Contains an excellent section on birefringence polarizers.

T-13 Twyman, F., *Optical glassworking* (Hilger and Watts, London, ed. 2, 1955), 288 pp. Contains an appendix on "Making polarizing prisms."

U-1 United States District Court for the District of Massachusetts. Civil Action No. 53-168. Opinion by Chief Justice Sweeney, issued Feb. 28, 1955.

V-1 Van de Hulst, H. C., *Light scattering by small particles* (Wiley, New York, 1957), 470 pp. Chapter 5 discusses Stokes parameters and matrices of Perrin, Mueller, and Jones.

V-2 Van Doorn, C. Z., and Y. Haven, "Dichroism of the F and M absorption bands in KCl," *Phys. Rev. 100*, 753 (1955).

V-3 Van Heel, A. C. S., "Interferometry with Savart's plate," appendix in J. Strong, S-31.

V-4 Vickers, A. E. J., "The polarizing microscope in organic chemistry and biology," *Research 9*, 67 (1956).

W-1 Wahlstrom, E. E., *Optical crystallography* (Wiley, New York, ed. 2, 1951).

W-2 Walker, J., *The analytical theory of light* (Cambridge University Press, Cambridge, England, 1904). Deals at length with many types of compensator.

W-3 Walker, M. J., "Matrix calculus and the Stokes parameters of polarized radiation," *Am. J. Phys. 22*, 170 (1954).

W-4 Waring, C. E., and R. L. Custer, "Determination of the Faraday effect," in A. Weissberger, ed., *Physical methods in organic chemistry*, vol. 1, part 3 (Interscience, New York, ed. 3, 1960), chap. 37.

W-5 Waterman, T. H., "Polarization patterns in submarine illumination," *Science 120*, 927 (1954).

W-6 Waterman, T. H., "Polarized light and animal navigation," *Sci. Am.* (July 1955), p. 88.

W-7 Waterman, T. H., and W. E. Westell, "Quantitative effect of the sun's position on submarine light polarization," *J. Marine Research* (*Sears Foundation*) *15*, 149 (1956).

W-8 Waterman, T. H., "Interaction of polarized light and turbidity in the orientation of *Daphnia* and *Mysidium*," *Z. vergleich. Physiol. 43*, 149 (1960).

W-9 Wayland, H., "Quantitative fluid flow visualization with streaming birefringence," *Phys. Rev. 98*, 255 (1955). Abstract.

W-10 Weber, G., "Polarization of the fluorescence of macromolecules," *Biochem. J. 51*, 145 (1952).

W-11 Weeks, D. W., "A study of sixteen coherency matrices," *J. Math. and Phys. 13*, 380 (1957).

W-12 Weigel, R. G., "Die Anwendung polarisierten Lichtes zur Verhinderung der Blendung im Kraftverkehr," *Optik 5*, 169 (1949).

W-13 Weigert, F., "New time phenomenon in photographic emulsions," *Trans. Faraday Soc. 34*, 927 (1938). Discusses photodichroism in gelatin emulsions that contain AgCl.

W-14 Weill, M. G., "Un appareil de mesure de la dépolarisation de la lumière diffusée," *J. phys. radium 18*, 78 S (1957). Employs Wollaston and Glazebrook polarizers in the photoelectric measurement of depolarization produced by liquids and gases.

W-15 Wellington, W. G., "Motor responses, etc., to plane polarized light," *Nature 172*, 1177 (1953).

W-16 West, C. D., "Crystallography of herapathite," *Am. Mineralogist 22*, 731 (1937).

W-17 West, C. D., "Polarizing accessories for microscopes," *J. Chem. Ed. 19*, 66 (1942).

W-18 West, C. D., "Structure-optical studies. I. X-ray diffraction by addition compounds of halogens with hydrophyllic organic polymers," *J. Chem. Phys. 15*, 689 (1947).

W-19 West, C. D., "On rendering surfaces anisotropic," *Glass Ind. 30*, 272 (May 1949). Discusses Beilby layer polarizers and work by Zocher.

W-20 West, C. D., and A. S. Makas, "The spectral dispersion of birefringence, especially of birefringent plastic sheets," *J. Opt. Soc. Amer. 39*, 791 (1949). Discusses achromatic retarders.

W-21 West, C. D., "Polariscopic and polarimetric examination of materials by transmitted light," in W. G. Berl, *Physical methods in chemical analysis* (Academic Press, New York, vol. 1, 1950), pp. 425–483.

W-22 West, C. C., and R. C. Jones, "On the properties of polarization elements as used in optical instruments. I. Fundamental considerations," *J. Opt. Soc. Amer. 41*, 976 (1951).

— West, C. D , U.S. patents:
2,420,273 (1947). Achromatic optical ring sight.
2,441,049 (1948). Achromatic retarder of organic plastic.

2,447,828 (1948). Prism-type polarizer employing isotropic block and thin birefringent sheet.

2,788,710 (1957). Electro-optical shutter employing cubic crystals of the class T_d, for example, cuprous halide.

W-23 Westfold, K. C., "New analysis of the polarization of radiation and the Faraday effect in terms of complex vectors," *J. Opt. Soc. Amer. 49*, 717 (1959).

W-24 White, C. T., "Polarized-light illumination of radar and sonar spaces and comparison with limited spectrum methods," Research and Development Report No. 669, U.S. Navy Electronics Laboratory, San Diego, California, Feb. 21, 1956; 12 pp.

— White, C. T., U.S. patent 2,793,361. Use of polarizers in combat information centers, etc.

W-25 Wilkinson, D. H., "A source of plane polarized gamma-rays of variable energy above 5.5 Mev," *Phil. Mag. 43*, 659 (1952). A degree of polarization of nearly 100 percent is achieved in the $^2H(p,\gamma)^3He$ reaction.

W-26 Wood, R. W., *Physical optics* (Macmillan, New York, ed. 3, 1934).

W-27 Worthing, A. G., "Deviation from lambert's law and polarization of light emitted by incandescent tungsten, tantalum, and molybdenum, and changes in the optical constants of tungsten with temperature," *J. Opt. Soc. Amer. 13*, 635 (1926).

W-28 Wright, F. E., "A spherical projection chart for use in the study of elliptically polarized light," *J. Opt. Soc. Amer. 20*, 529 (1930). Uses Poincaré sphere.

W-29 Wright, N., "A transmitting polarizer for infra-red radiation," *J. Opt. Soc. Amer. 38*, 69 (1948).

Y-1 Yamaguti, T., "On a sodium nitrate polarization plate of scattering type," *J. Opt. Soc. Amer. 45*, 891 (1955).

Y-2 Young, T., "Miscellaneous Works," Vol. 1, 1855.

Z-1 Zandman, F., and M. R. Wood, "Photostress: a new technique for photoelastic stress analysis for observing and measuring surface strains on actual structures and parts," *Product Eng. 27*, 167 (1956).

Z-2 Zandman, F., "Make strain visible with photostress analysis," *Product Eng. 30*, 43 (1959).

Z-3 Zarem, A. M., F. R. Marshall, and S. M. Hauser, "Millimicrosecond Kerr cell camera shutter," *Rev. Sci. Instr. 29*, 1041 (1958). Describes shutter having an exposure time of 5 mμs.

— Zeiss (Carl Zeiss, Inc.), German patent 1,015,236 (1957). Infrared dichroic polarizer by Drechsel.

Z-4 Zimm, B. H., "Photoelectric flow birefringence instrument of high sensitivity," *Rev. Sci. Instr. 29*, 360 (1958).

Z-5 Zimmern, A., Belgian and Austrian patents mentioned in U-1. An article entitled "Sur une nouvelle méthode de production de l'hérapathite," *Compt. rend. 182*, 1082 (1926). These papers deal with methods for producing large area, crystalline, polarizing layers on specially prepared surfaces.

Z-6 Zocher, H., and F. C. Jacoby, "Über die optische Anisotropie selektiv absorbierender Farbstoffe," *Kolloidchem. Beih. 24*, 365 (1927). Tabulates dichroism data on about 100 organic dyes. Discusses streaming dichroism.

Z-7 Zocher, H., and K. Coper, "Über die Erzeugung der Anisotropie von

Oberflächen," *Z. physik. Chem.* *132*, 295 (1928). Describes polarizers made by rubbing a glass surface and applying methylene blue. (Translated and presented with W-19 by West.)

Z-8 Zocher, H., and K. Coper, "Über die durch den Weigerteffekt in Photochlorid erzeugte Anisotropie," *Z. physik. Chem.* *132*, 303 (1928).

Z-9 Zocher, H., and K. Coper, "Über die Erzeugung optischer Aktivität durch zirkulares Licht," *Z. physik. Chem.* *132*, 313 (1928).

INDEX